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Incentives to Work or Incentives to Quit?

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Introduction: Motivation

Employees and firms often learn about the quality of their match over time...and this learning influences separation decisions.

When match quality is firm-specific, the employer may capture some of the surplus of the employment relation.

Questions: What contracts do firms offer? How do they affect profits and compensation? What contracts should be offered?.

Introduction: Preview of Results

1. Even conditional on (best belief of) match quality, total tenure is informative about dynamics of productivity and compensation.

2. Profits depend not only on effort but also on firm experimentation with the workforce which determines the quality mix of workers and turnover.

3. A two-step procedure can be used to approximate the value of continued employment of the worker.

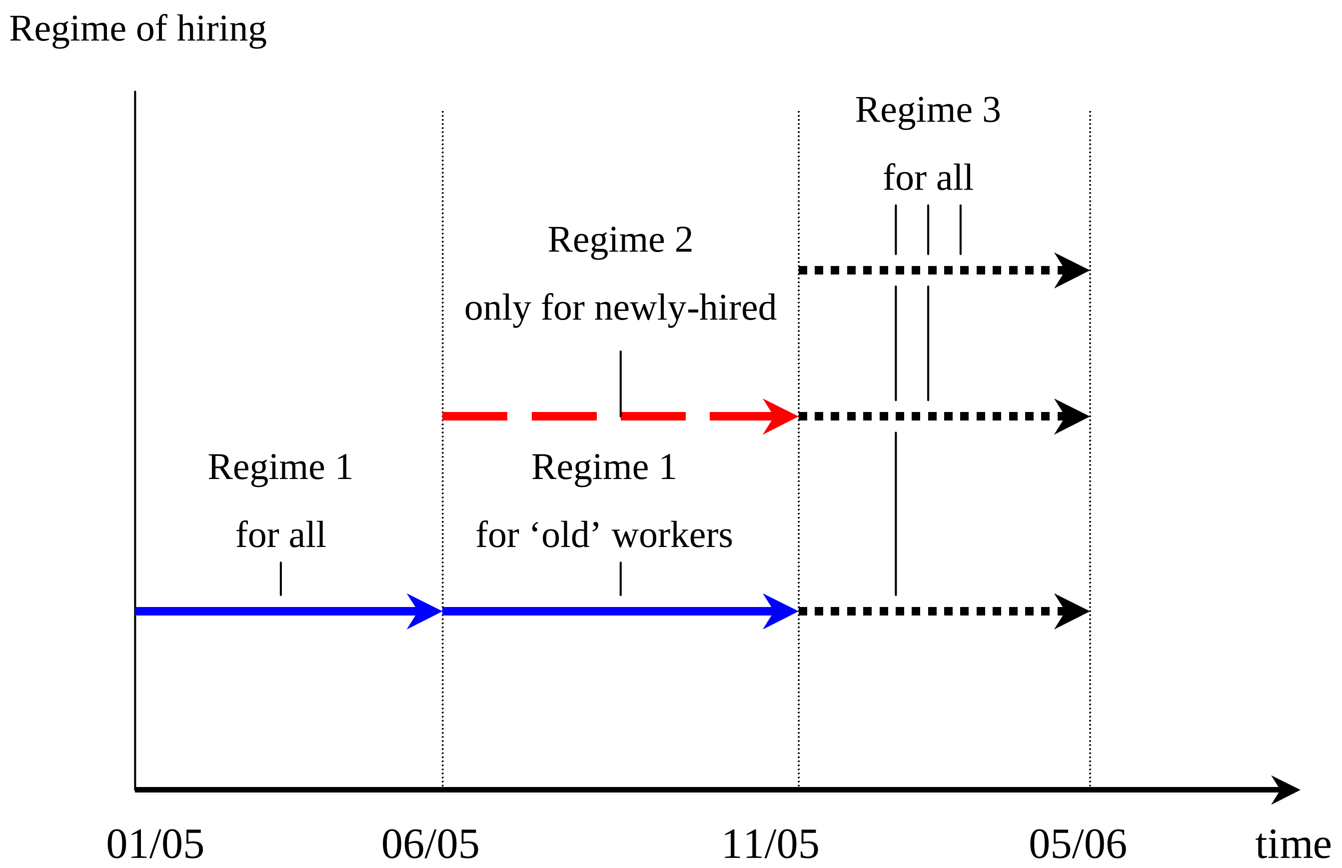
Data: Description

Come from a call center that collects outstanding debt on behalf of cable TV companies. Main features are:

- Objective measure of performance: calls that end with debt collection per hour;

- Known pay policies: quasi-experimental variation in pay regimes, all based on hourly pay plus a bonus proportionate to performance;

-Turnover: more than 50% of employees quit within first 6 months across regimes.



Regime of hiring
Regime 3
for all
Regime 2
only for newly-hired
Regime 1
for all
Regime 1
for 'old' workers
01/05
06/05
11/05
05/06
time

Model: Description

Each month, the worker draws an outside offer $\xi_t \sim N\left(0, \sigma^2_{\xi^*}\right)$and decides to stay or quit.

If the worker stays, she chooses optimal effort l_t.

Then, she observed the performance signal y_t...

and forms a new posterior belief θ_t

The firm commits to a compensation policy R.

Employees are free to leave at the beginning of each period.

Model: Technology

The data impose strong restrictions on the functional form of y_t. The following specification is consistent with these restrictions:

$$y_t = \theta + l_t + g(t) + \varepsilon_t$$

where $\theta_i \sim N\left(\theta, \sigma_\theta^2\right)$ is ability, t tenure, $g(t)$ experience, l_t effort, and $\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$ noise.

Assume that $\xi_t, \varepsilon_t, \theta$ are iid. If prior at $t = 1$ is $N\left(0, \sigma_\theta^2\right)$, posterior belief at $t > 1$ is $\theta_t \sim N\left(\mu_t, \sigma_t^2\right)$ and (μ_t, t) is sufficient statistic.

Model: Compensation and Utility

Let the wage policy under R after history Y_t be $W\left(Y_t, t\right)$ .The VNM utility is:

$$E\left[W\left(Y_t, t\right)\right] - \frac{\gamma}{1+\frac{1}{\psi}} l_t^{1+\frac{1}{\psi}}$$

Considered policies: ensure optimal effort by selling current output to worker:

$\longrightarrow$ primary focus on valuation of match when firm can rehire;

$\longrightarrow$optimal effort is time-invariant l if effort is observable but not verifiable.

Model: Worker's Problem

Denote expected utility $U(R, \mu_t, t)$. The problem of an employed worker is:

$$V(R, \mu_t, t) = U(R, \mu_t, t) + \delta E\left[\max(\xi, V\left(R, \mu_{t+1}, t+1\right)\right]$$

with E over outside offers and posterior beliefs at $t+1$ given information at t. Employed at t if:

$$V(\mu_t, R_t, t) - \xi_t > 0$$

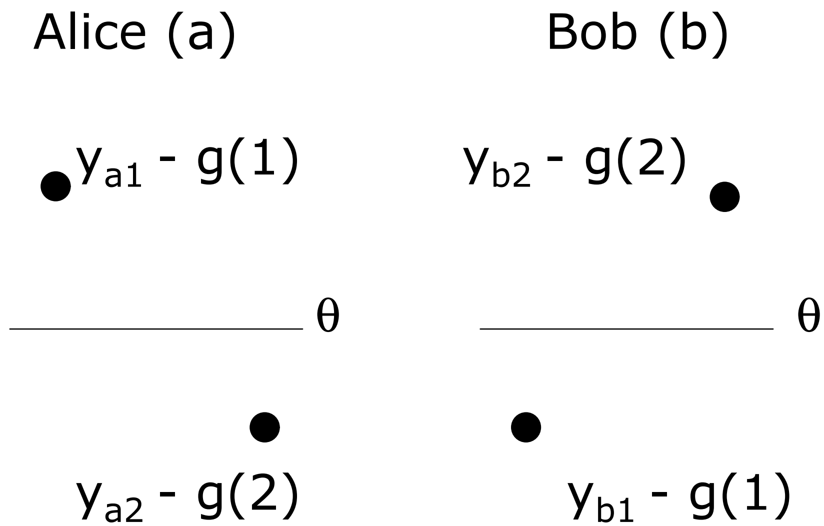
Model: Firm's Problem

Revenue: r. Turnover cost: c. Quitting workers are immediately replaced. Technology: constant returns to scale.

Let profit at t conditional on staying and (μ_t, t) be $\pi(R, \mu_t, t)$, probability of staying $\geq t$ be $p_s(\mu_t, t, R)$ and probability of quitting at t $p_q(\mu_t, t, R)$. Total profit per workstation is:

$$\pi(R) = E\left\{\sum_{t=1}^{\infty} \tilde{\delta}^{t-1}\left[p_t(\mu_t, t, R)\,\pi(\mu_t, t, R) + p_q(\mu_t, t, R)\,(\pi(R) - c)\right]\right\}$$

$\mathbf{E}(\varepsilon_k | stay > t,\ \theta) > \mathbf{0}$: Alice and Bob

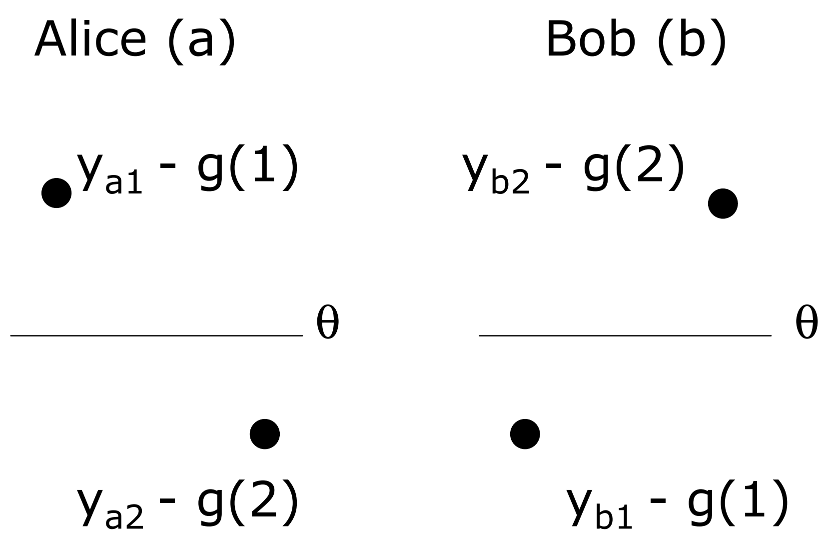


Performance: $y_{it} = \theta + g(t) + \varepsilon_{it},\ i = a, b,\ t = 1, 2$

Equal Signals: $y_{a1} - g(1) = y_{b2} - g(2),\ y_{a2} - g(2) = y_{b1} - g(1)$

Payoff: y_{i2}

Beliefs affect quit decision, leading to...

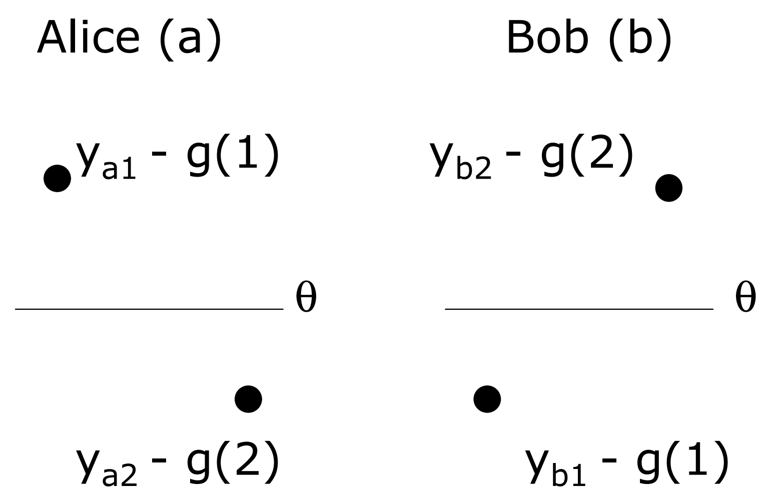


After first signal: i quits if $E\left(y_{i1} + y_{i2} | y_{i1}\right) < \xi_{i1}$, where ξ_{i1} is realized outside offer; $\xi_{i1} \perp \varepsilon_{it}$

Known ability: $\Pr\left(\text{Alice quits after } t = 1\right) = \Pr\left(\text{Bob quits after } t = 1\right)$

Learning about ability: $\Pr\left(\text{Alice quits after } t = 1\right) < \Pr\left(\text{Bob quits after } t = 1\right)$

...Correlation between decision to stay and noise ε_{i1}.



Learning about Ability: More "Alices" than "Bobs" observe both signals $\rightarrow$ change in performance not related to experience.

Estimation: First Step

Estimate the following attrition model using ML:

$$y_t = \theta + l(R) + g(t) + \varepsilon_t$$

$$s_k = 1\left[H(\mu_k, R, k) - \xi_k > 0\right]$$

where y_t, $t > 1$, is observed if $s_k = 1$ for all $k = 1, ..., t$, $\xi_t \sim N(0, 1)$. $H(\mu_k, R, k)$ is approximated using a linear combination of orthogonal polynomials of the explanatory variables.

Estimation: Second Step

Let the difference in effort under regimes 1 and 2 be Δl. From the performance equation,

$$\Delta l = \left(\frac{1}{\gamma}\right)^{\psi} \left(\beta_1^{\psi} - \beta_2^{\psi}\right) => \gamma\left(\psi, \widehat{\Delta l}\right)$$

To save on notation, define

$$\lambda\left(H\left(\mu_t, R_t, t\right)\right) = E_{\xi} \max\left\{\xi, H\left(\mu_k, R, k\right)\right\}$$
$$= H\left(\mu_k, R, k\right).\Phi\left(H\left(\mu_k, R, k\right)\right) + \varphi\left(H\left(\mu_k, R, k\right)\right)$$

Estimation: Second Step (contd.)

From the definition of $V(\mu_{it}, R_{it}, t)$

$$H(\mu_t, R_t, t) = U(R, \mu_t, t) + \delta\left[E(\lambda\left(H\left(\mu_{t+1}, R, t+1\right)\right)\right]$$

$\Rightarrow$ condition $E(M_t(\Theta_2)) = 0$ where Θ_2 is the vector of remaining unknown parameters. Stacking all such moment conditions into $M(\Theta_2)$, solve

$$\min_{\Theta_2} (M(\Theta_2))' \, \Omega^{-1} (M(\Theta_2))$$

Results: Structural Estimates

Parameter:	Coefficient.	Std. Err.
ψ	3.24	0.20
γ	3.92	0.23
δ	0.76	0.10
Δl	-0.21	0.06
Δdisutility	-0.65	0.07
experience by $t = 6$	1.02	0.09
σ_θ^2	0.48	0.02
μ_θ	2.02	0.11
χ_{12}^2 test stat.	6.14	

Linear Contract

Revenue per successful call is \$8.5.

Turnover cost is \$750. The firm immediately hires a replacement when one quits.

Consider a linear contract in performance: $w = \alpha_w + \beta_w y$.Solve for the optimal contract numerically.

Trade-offs: **(1)** rewarding effort and keeping high match quality workers; **(2)** selecting high quality employees on the job; **(3)** experimenting with new workers.

Linear Contract (contd.)

Pay policy:	α	β	l	$E(t)$	$E(\theta)$	π
hourly wage, $9.5	9.5	0	0	3.22	2.03	19.45
regime 1	3.8	3.3	**0.59**	**11.3**	**2.88**	**167.81**
regime 2	3.5	2.8	0.38	6.23	2.83	110.17
regime, optimal	3.65	3.24	**0.55**	**9.85**	**2.93**	**174.24**

Result: The optimal pay regime is very close to the original regime 1. The turnover channel is more important for profits than the effort choice channel.

Contracts in Current and Past Performance (contd.)

Consider contracts for period t of (y_t, μ_t) but do not vary over t.

$\longrightarrow$ as above, but allow the use of past information.

Then extend to contracts for period t that are function of (y_t, μ_t, t) .

$\longrightarrow$ as above, but allow for a contract that changes with the precision of beliefs.

Contracts in Current and Past Performance (contd.)

1. Sell the contract: provide maximum incentives on current output. (intuition: sell the contract)

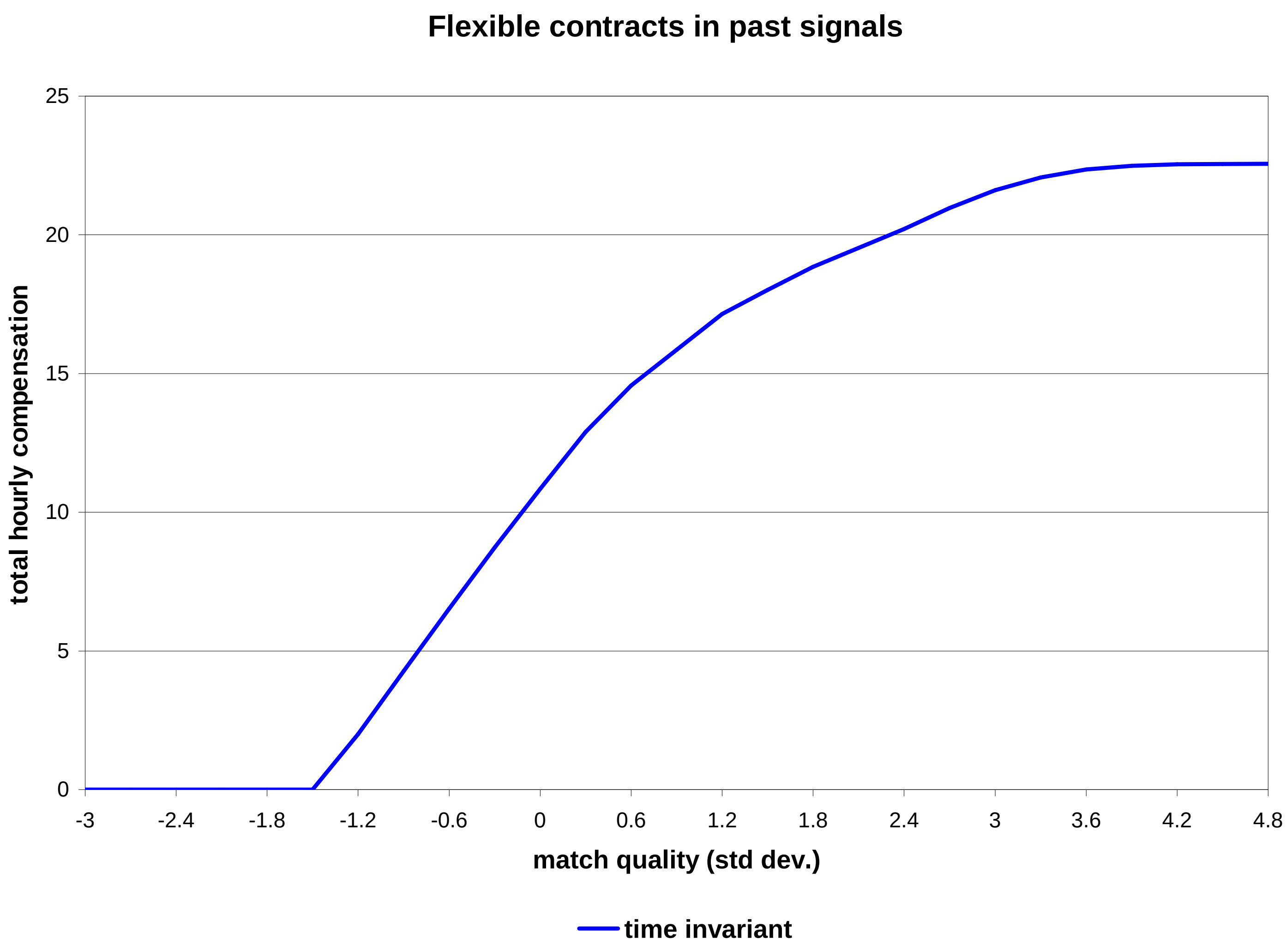
2. Compensation increases at decreasing rate in μ_t

3. Option value for the worker and for the firm decrease with t. As t increases, compensation schedule:

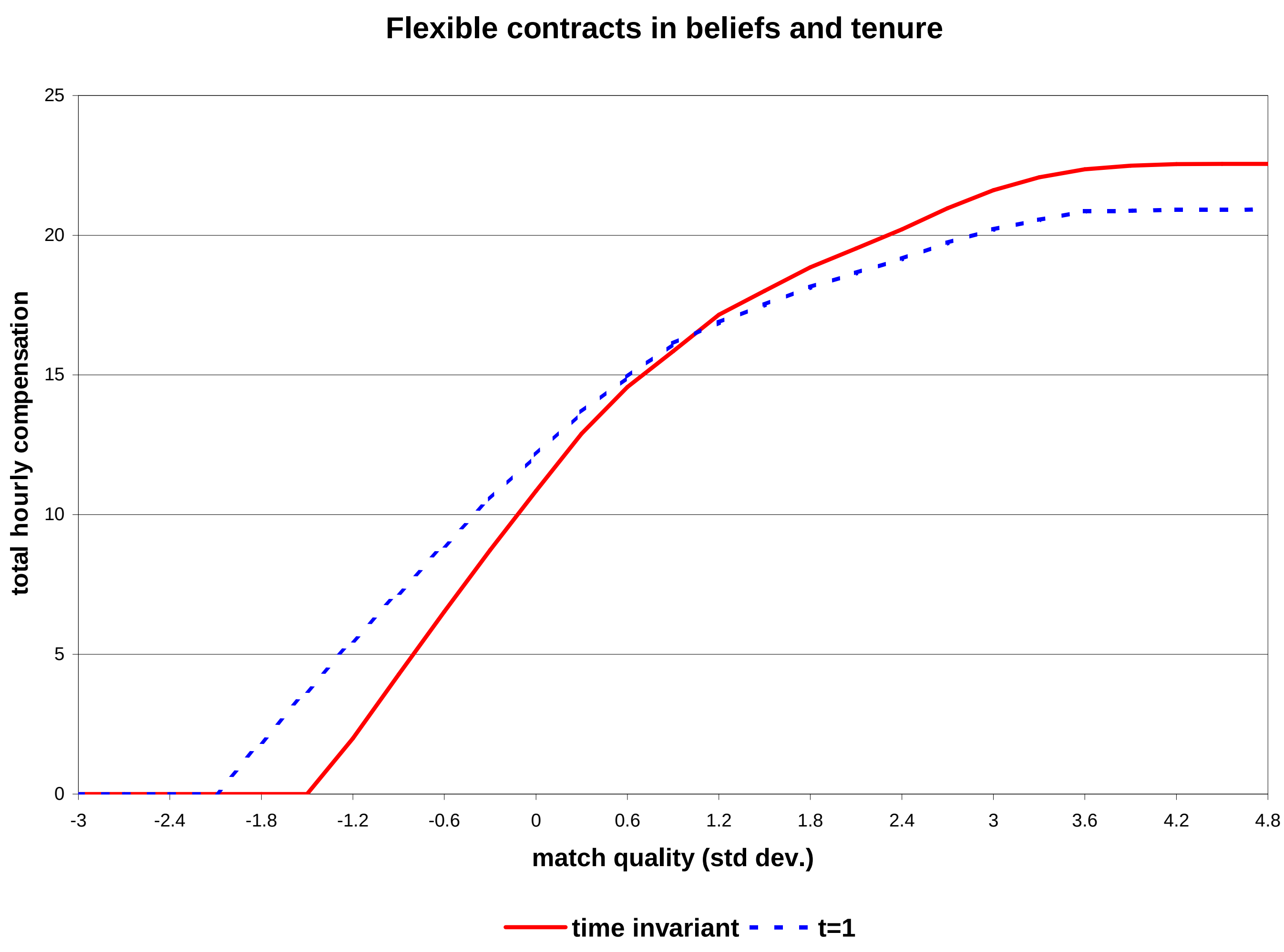
$\longrightarrow$ shifts to the right and becomes steeper.

Contracts in Current and Past Performance

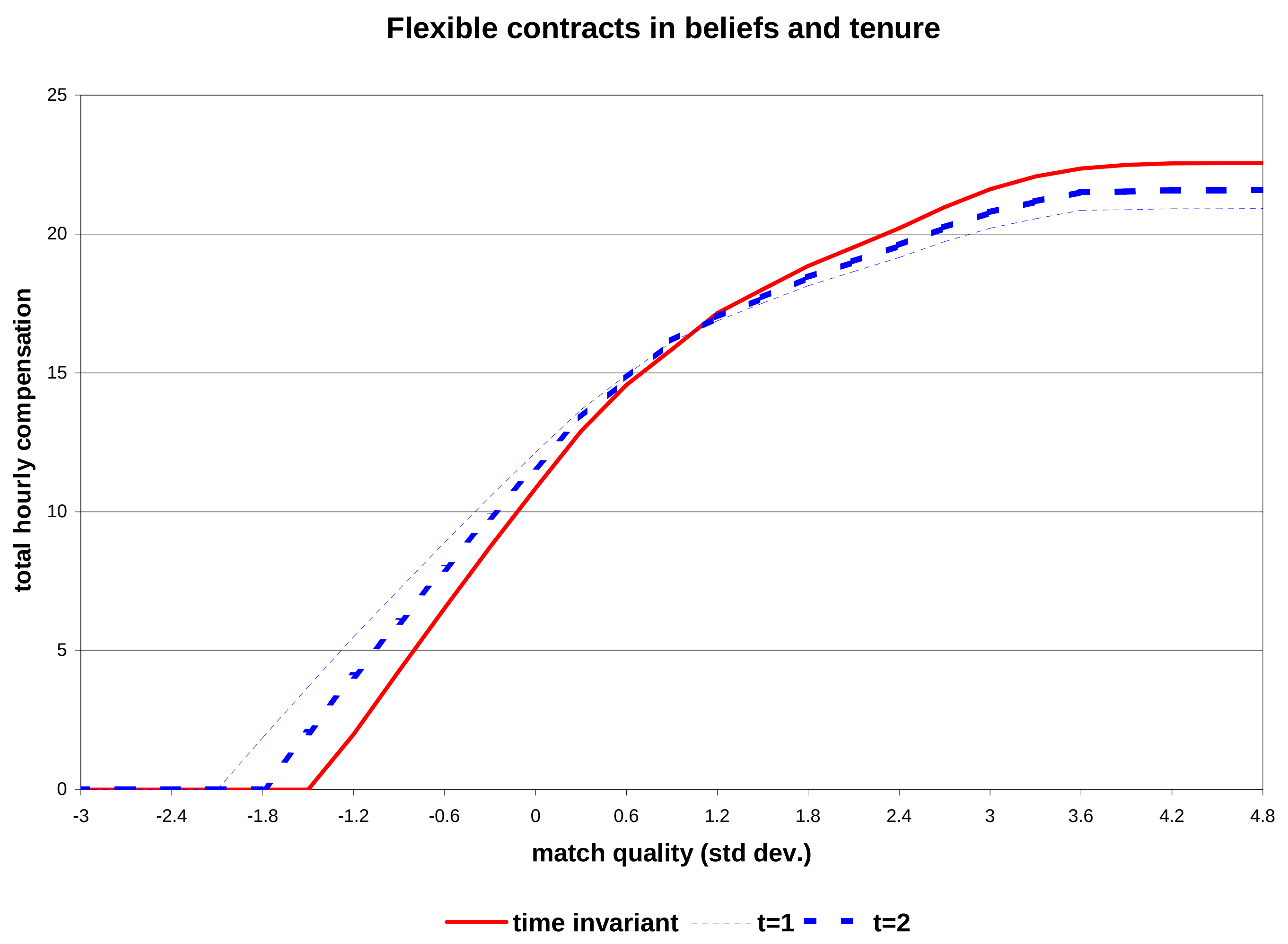
Pay policy:	l	$E(t)$	$E(\theta)$	π
regime, time invariant	**5.23**	**8.53**	**3.23**	**256.84**
regime, varying with t	**5.23**	**9.12**	**3.46**	**272.65**
regime, optimal linear	**0.55**	**9.85**	**2.93**	**174.24**



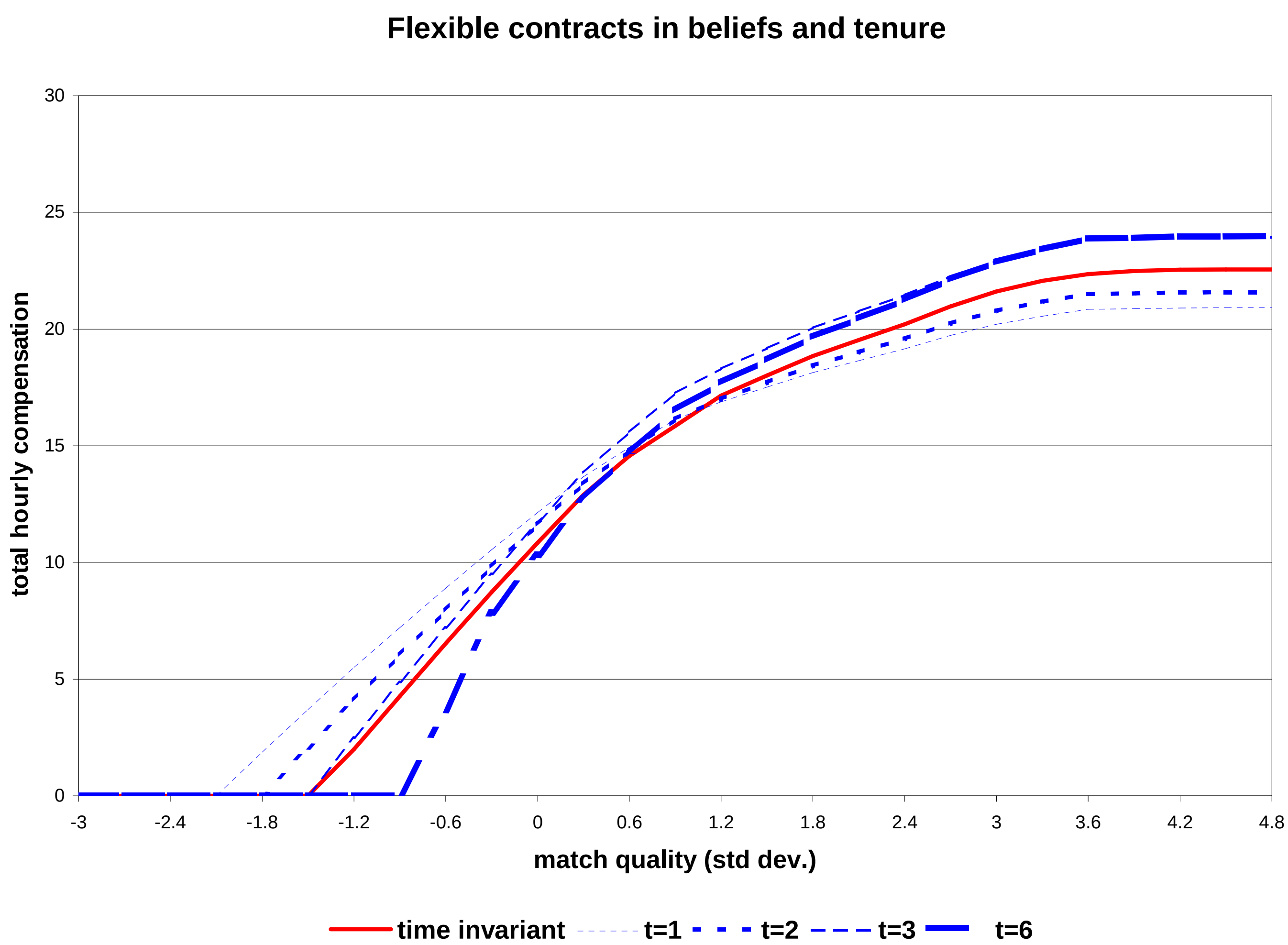
Flexible contracts in past signals
25
20
15
10
5
0
-3
-2.4
-1.8
-1.2
-0.6
0
0.6
1.2
1.8
2.4
3
3.6
4.2
4.8
total hourly compensation
match quality (std dev.)
time invariant

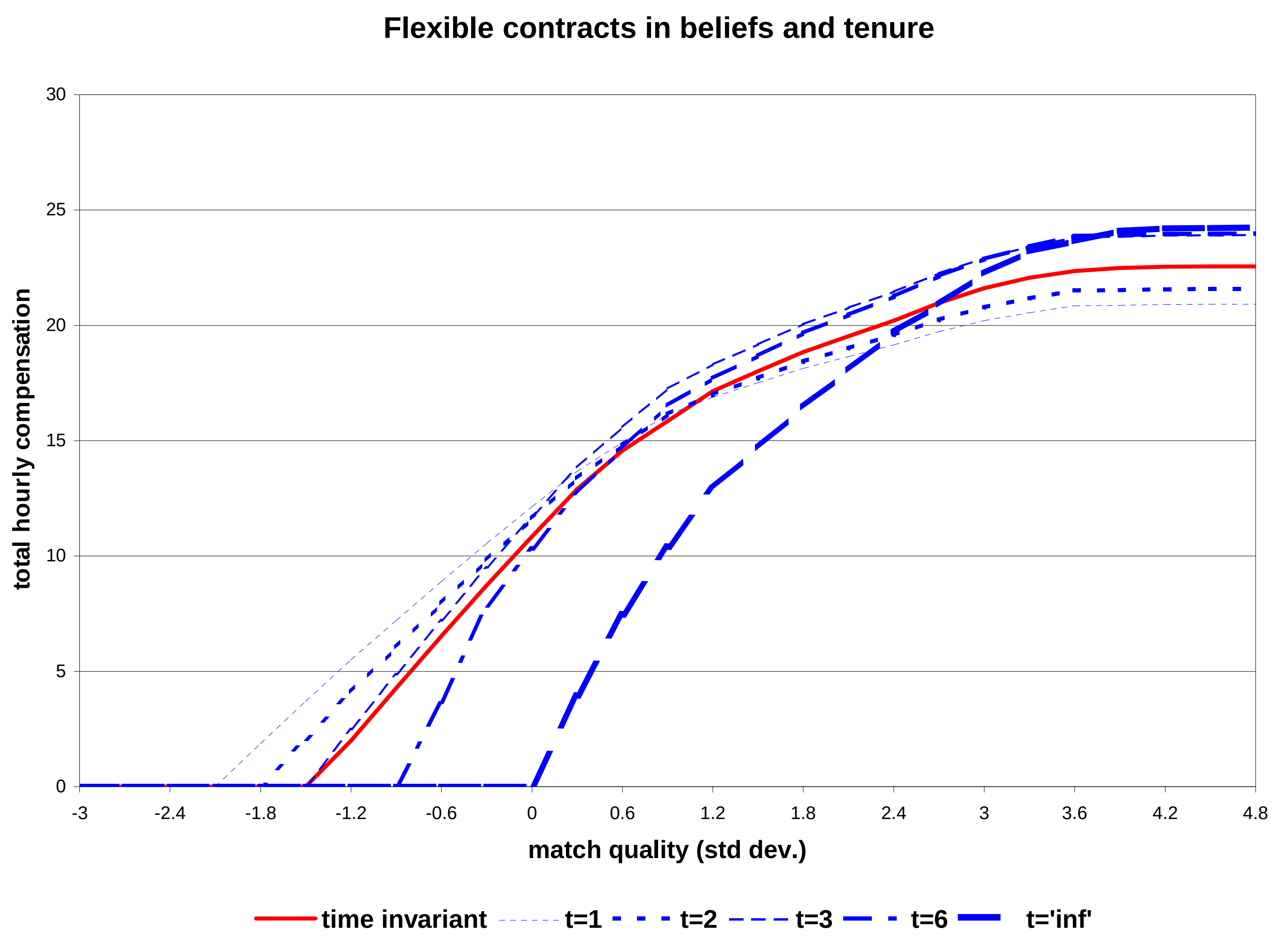


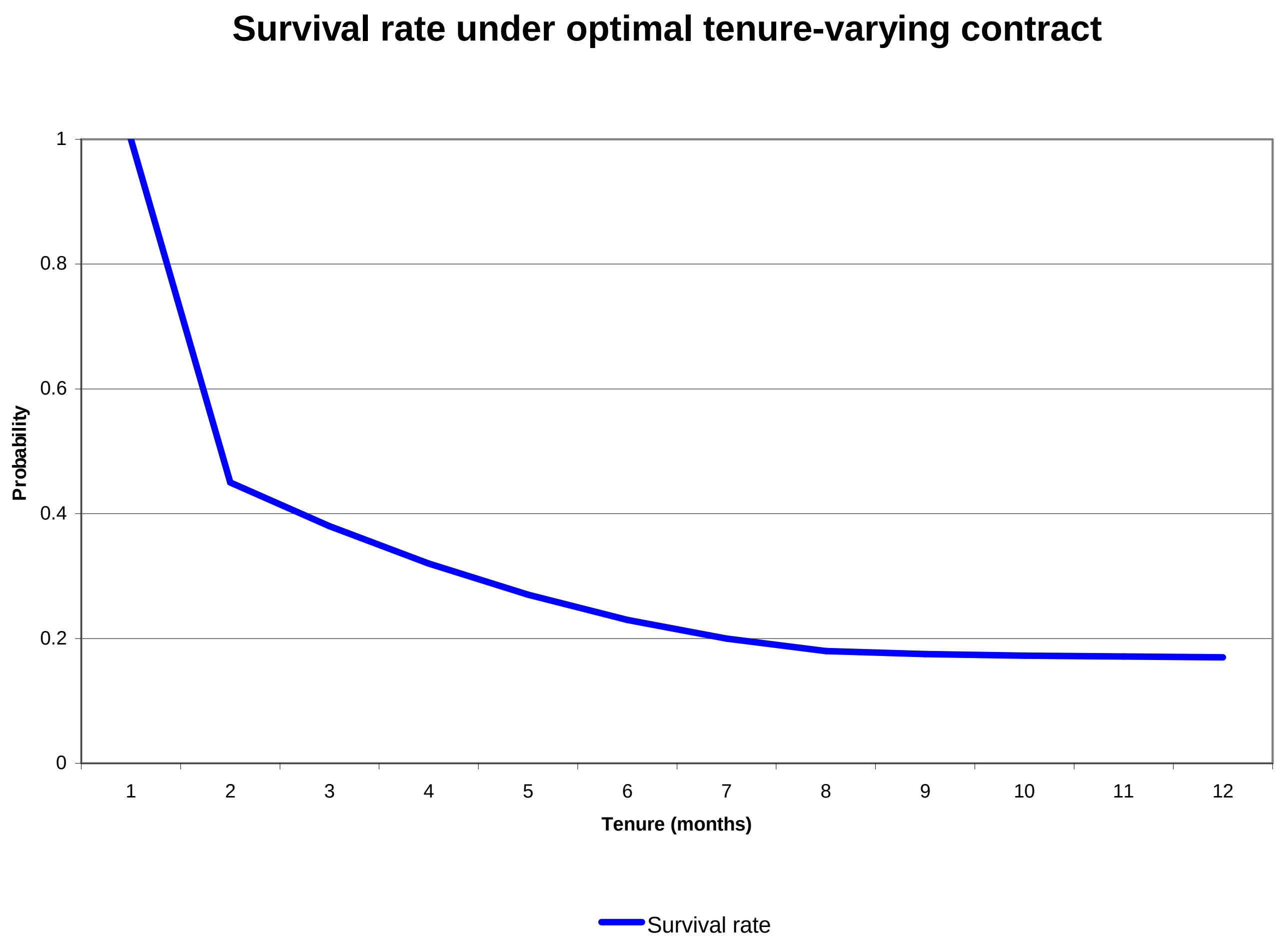
Flexible contracts in beliefs and tenure
total hourly compensation
0
5
10
15
20
25
-3
-2.4
-1.8
-1.2
-0.6
0
0.6
1.2
1.8
2.4
3
3.6
4.2
4.8
match quality (std dev.)
time invariant
t=1

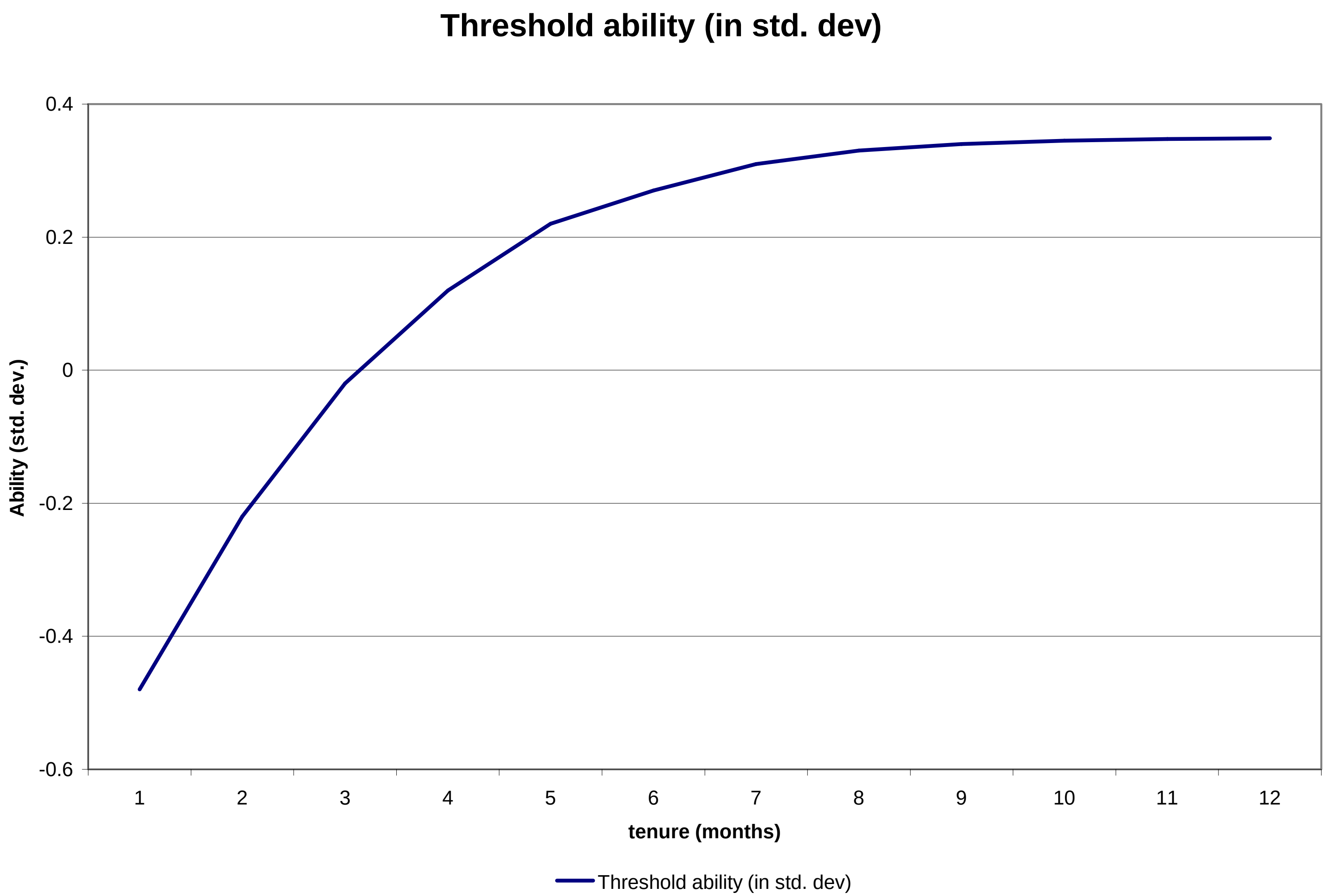


Flexible contracts in beliefs and tenure
total hourly compensation
0
5
10
15
20
25
-3
-2.4
-1.8
-1.2
-0.6
0
0.6
1.2
1.8
2.4
3
3.6
4.2
4.8
match quality (std dev.)
time invariant
t=1
t=2


Flexible contracts in beliefs and tenure
total hourly compensation
0
5
10
15
20
25
30
-3
-2.4
-1.8
-1.2
-0.6
0
0.6
1.2
1.8
2.4
3
3.6
4.2
4.8
match quality (std dev.)
time invariant
t=1
t=2
t=3
t=6


Flexible contracts in beliefs and tenure
total hourly compensation
30
25
20
15
10
5
0
-3
-2.4
-1.8
-1.2
-0.6
0
0.6
1.2
1.8
2.4
3
3.6
4.2
4.8
match quality (std dev.)
time invariant
t=1
t=2
t=3
t=6
t='inf'


Survival rate under optimal tenure-varying contract
Probability
1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
6
7
8
9
10
11
12
Tenure (months)
Survival rate


Threshold ability (in std. dev)
0.4
0.2
0
-0.2
-0.4
-0.6
Ability (std. dev.)
1
2
3
4
5
6
7
8
9
10
11
12
tenure (months)
Threshold ability (in std. dev)

Conclusion

1. Presented and analyzed a model of learning about match quality with rehiring.

2. Found the optimal contract and characterized the value of experimentation.

3. Showed how to estimate easily structural models with Bayesian learning.