Incentives to Work or Incentives to Quit?

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Introduction: Motivation

Employees and firms often learn about the quality of their match over time...and this learning influences separation decisions.

When match quality is firm-specific, the employer may capture some of the surplus of the employment relation.

**Questions:** What contracts do firms offer? How do they affect profits and compensation? What contracts should be offered?
Introduction: Preview of Results

1. Even conditional on (best belief of) match quality, total tenure is informative about dynamics of productivity and compensation.

2. Profits depend not only on effort but also on firm experimentation with the workforce which determines the quality mix of workers and turnover.

3. A two-step procedure can be used to approximate the value of continued employment of the worker.
Data: Description

Come from a call center that collects outstanding debt on behalf of cable TV companies. Main features are:

- Objective measure of performance: calls that end with debt collection per hour;

- Known pay policies: quasi-experimental variation in pay regimes, all based on hourly pay plus a bonus proportionate to performance;

- Turnover: more than 50% of employees quit within first 6 months across regimes.
Regime of hiring

- Regime 1: for all
- Regime 2: only for newly-hired
- Regime 1: for ‘old’ workers
- Regime 3: for all
Model: Description

Each month, the worker draws an outside offer $\xi_t \sim N \left(0, \sigma_{\xi}^2\right)$ and decides to stay or quit.

If the worker stays, she chooses optimal effort $l_t$.

Then, she observed the performance signal $y_t$...

and forms a new posterior belief $\theta_t$

The firm commits to a compensation policy $R$.

Employees are free to leave at the beginning of each period.
Model: Technology

The data impose strong restrictions on the functional form of $y_t$. The following specification is consistent with these restrictions:

$$y_t = \theta + l_t + g(t) + \varepsilon_t$$

where $\theta^i \sim N\left(\theta, \sigma^2_\theta\right)$ is ability, $t$ tenure, $g(t)$ experience, $l_t$ effort, and $\varepsilon_t \sim N\left(0, \sigma^2_\varepsilon\right)$ noise.

Assume that $\xi_t, \varepsilon_t, \theta$ are iid. If prior at $t = 1$ is $N\left(0, \sigma^2_\theta\right)$, posterior belief at $t > 1$ is $\theta_t \sim N\left(\mu_t, \sigma^2_t\right)$ and $(\mu_t, t)$ is sufficient statistic.
Model: Compensation and Utility

Let the wage policy under $R$ after history $Y_t$ be $W(Y_t, t)$. The VNM utility is:

$$E[W(Y_t, t)] - \frac{\gamma}{1 + \frac{1}{\psi}} l_t^{1+\frac{1}{\psi}}$$

Considered policies: ensure optimal effort by selling current output to worker:

→ primary focus on valuation of match when firm can rehire;

→ optimal effort is time-invariant $l$ if effort is observable but not verifiable.
Model: Worker’s Problem

Denote expected utility $U(R, \mu_t, t)$. The problem of an employed worker is:

$$V(R, \mu_t, t) = U(R, \mu_t, t) + \delta E\left[\max(\xi, V(R, \mu_{t+1}, t+1)\right]$$

with $E$ over outside offers and posterior beliefs at $t + 1$ given information at $t$. Employed at $t$ if:

$$V(\mu_t, R_t, t) - \xi_t > 0$$
Model: Firm’s Problem

Revenue: $r$. Turnover cost: $c$. Quitting workers are immediately replaced. Technology: constant returns to scale.

Let profit at $t$ conditional on staying and $(\mu_t, t)$ be $\pi(R, \mu_t, t)$, probability of staying $\geq t$ be $p_s(\mu_t, t, R)$ and probability of quitting at $t$ $p_q(\mu_t, t, R)$. Total profit per workstation is:

$$\pi(R) = E \left\{ \sum_{t=1}^{\infty} \delta^{t-1} [p_t(\mu_t, t, R) \pi(\mu_t, t, R) + p_q(\mu_t, t, R) (\pi(R) - c)] \right\}$$
\[ E(\varepsilon_k | \text{stay } > t, \theta) > 0: \text{ Alice and Bob} \]

Alice (a)          Bob (b)

\[ \bullet y_{a1} - g(1) \quad y_{b2} - g(2) \bullet \]

\[ \theta \quad \theta \]

\[ y_{a2} - g(2) \quad y_{b1} - g(1) \]

Performance: \[ y_{it} = \theta + g(t) + \varepsilon_{it}, i = a, b, t = 1, 2 \]

Equal Signals: \[ y_{a1} - g(1) = y_{b2} - g(2), y_{a2} - g(2) = y_{b1} - g(1) \]

Payoff: \( y_{i2} \)
Beliefs affect quit decision, leading to...

Alice (a)  Bob (b)

\[ y_{a1} - g(1) \]
\[ y_{a2} - g(2) \]
\[ \theta \]
\[ y_{b1} - g(1) \]
\[ y_{b2} - g(2) \]
\[ \theta \]

After first signal: \( i \) quits if \( E (y_{i1} + y_{i2} | y_{i1}) < \xi_{i1} \), where \( \xi_{i1} \) is realized outside offer; \( \xi_{i1} \perp \epsilon_{it} \)

**Known ability:** \( \Pr (Alice \text{ quits after } t = 1) = \Pr (Bob \text{ quits after } t = 1) \)

**Learning about ability:** \( \Pr (Alice \text{ quits after } t = 1) < \Pr (Bob \text{ quits after } t = 1) \)
...Correlation between decision to stay and noise $\varepsilon_{i1}$.

Learning about Ability: More "Alices" than "Bobs" observe both signals $\rightarrow$ change in performance not related to experience.
Estimation: First Step

Estimate the following attrition model using ML:

\[ y_t = \theta + l(R) + g(t) + \varepsilon_t \]

\[ s_k = 1 [H (\mu_k, R, k) - \xi_k > 0] \]

where \( y_t, t > 1 \), is observed if \( s_k = 1 \) for all \( k = 1, ..., t, \xi_t \sim N (0, 1) \). \( H (\mu_k, R, k) \) is approximated using a linear combination of orthogonal polynomials of the explanatory variables.
Estimation: Second Step

Let the difference in effort under regimes 1 and 2 be $\Delta l$. From the performance equation,

$$\Delta l = \left( \frac{1}{\gamma} \right)^\psi (\beta_1^\psi - \beta_2^\psi) =\gamma (\psi, \Delta l)$$

To save on notation, define

$$\lambda(H(\mu_t, R_t, t)) = E_{\xi} \max \{\xi, H(\mu_k, R, k)\}$$

$$= H(\mu_k, R, k) \cdot \Phi(H(\mu_k, R, k)) + \varphi(H(\mu_k, R, k))$$
Estimation: Second Step (contd.)

From the definition of $V(\mu_{it}, R_{it}, t)$

$$H(\mu_t, R_t, t) = U(R, \mu_t, t) + \delta [E(\lambda(H(\mu_{t+1}, R, t + 1)))]$$

⇒ condition $E(M_t(\Theta_2)) = 0$ where $\Theta_2$ is the vector of remaining unknown parameters. Stacking all such moment conditions into $M(\Theta_2)$, solve

$$\min_{\Theta_2} (M(\Theta_2))' \Omega^{-1} (M(\Theta_2))$$
## Results: Structural Estimates

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Coefficient</th>
<th>Std. Err.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\psi$</td>
<td>3.24</td>
<td>0.20</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>3.92</td>
<td>0.23</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.76</td>
<td>0.10</td>
</tr>
<tr>
<td>$\Delta l$</td>
<td>-0.21</td>
<td>0.06</td>
</tr>
<tr>
<td>$\Delta \text{disutility}$</td>
<td>-0.65</td>
<td>0.07</td>
</tr>
<tr>
<td>$\text{experience by } t = 6$</td>
<td>1.02</td>
<td>0.09</td>
</tr>
<tr>
<td>$\sigma^2_\theta$</td>
<td>0.48</td>
<td>0.02</td>
</tr>
<tr>
<td>$\mu_\theta$</td>
<td>2.02</td>
<td>0.11</td>
</tr>
<tr>
<td>$\chi^2_{12}$ test stat.</td>
<td></td>
<td>6.14</td>
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</tbody>
</table>
Linear Contract

Revenue per successful call is $8.5.

Turnover cost is $750. The firm immediately hires a replacement when one quits.

Consider a linear contract in performance: \( w = \alpha w + \beta w y \). Solve for the optimal contract numerically.

Trade-offs: (1) rewarding effort and keeping high match quality workers; (2) selecting high quality employees on the job; (3) experimenting with new workers.
Linear Contract (contd.)

<table>
<thead>
<tr>
<th>Pay policy:</th>
<th>$\alpha$</th>
<th>$\beta$</th>
<th>$l$</th>
<th>$E(t)$</th>
<th>$E(\theta)$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>hourly wage, $9.5$</td>
<td>9.5</td>
<td>0</td>
<td>0</td>
<td>3.22</td>
<td>2.03</td>
<td>19.45</td>
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<tr>
<td>regime 1</td>
<td>3.8</td>
<td>3.3</td>
<td>0.59</td>
<td>11.3</td>
<td>2.88</td>
<td>167.81</td>
</tr>
<tr>
<td>regime 2</td>
<td>3.5</td>
<td>2.8</td>
<td>0.38</td>
<td>6.23</td>
<td>2.83</td>
<td>110.17</td>
</tr>
<tr>
<td>regime, optimal</td>
<td>3.65</td>
<td>3.24</td>
<td>0.55</td>
<td>9.85</td>
<td>2.93</td>
<td>174.24</td>
</tr>
</tbody>
</table>

**Result:** The optimal pay regime is very close to the original regime 1. The turnover channel is more important for profits than the effort choice channel.
Contracts in Current and Past Performance (contd.)

Consider contracts for period $t$ of $(y_t, \mu_t)$ but do not vary over $t$.

→ as above, but allow the use of past information.

Then extend to contracts for period $t$ that are function of $(y_t, \mu_t, t)$.

→ as above, but allow for a contract that changes with the precision of beliefs.
Contracts in Current and Past Performance (contd.)

1. Sell the contract: provide maximum incentives on current output. (intuition: sell the contract)

2. Compensation increases at decreasing rate in $\mu_t$

3. Option value for the worker and for the firm decrease with $t$. As $t$ increases, compensation schedule:

   $\rightarrow$ shifts to the right and becomes steeper.
## Contracts in Current and Past Performance

<table>
<thead>
<tr>
<th>Pay policy:</th>
<th>$l$</th>
<th>$E(t)$</th>
<th>$E(\theta)$</th>
<th>$\pi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>regime, time invariant</td>
<td>5.23</td>
<td>8.53</td>
<td>3.23</td>
<td>256.84</td>
</tr>
<tr>
<td>regime, varying with $t$</td>
<td>5.23</td>
<td>9.12</td>
<td>3.46</td>
<td>272.65</td>
</tr>
<tr>
<td>regime, optimal linear</td>
<td>0.55</td>
<td>9.85</td>
<td>2.93</td>
<td>174.24</td>
</tr>
</tbody>
</table>
Flexible contracts in past signals

![Graph showing the relationship between match quality (std dev.) and total hourly compensation. The line is labeled as time invariant.]
Flexible contracts in beliefs and tenure

![Graph showing the relationship between total hourly compensation and match quality (std dev.)]

The graph illustrates the impact of flexible contracts on beliefs and tenure. The x-axis represents match quality (standard deviation), while the y-axis shows total hourly compensation.

Legend:
- Red line: time invariant
- Dashed blue line: t=1
- Solid blue line: t=2
Flexible contracts in beliefs and tenure

![Graph showing flexible contracts in beliefs and tenure. The graph plots total hourly compensation against match quality (standard deviation) for different time periods (t=1, t=2, t=3, t=6). The graph highlights the impact of time-invariant and time-varying factors on compensation, illustrating how beliefs and tenure influence outcomes.]
Flexible contracts in beliefs and tenure

match quality (std dev.)
total hourly compensation
time invariant t=1 t=2 t=3 t=6 t='inf'

-3 -2.4 -1.8 -1.2 -0.6 0 0.6 1.2 1.8 2.4 3 3.6 4.2 4.8
Survival rate under optimal tenure-varying contract

![Survival rate graph](image-url)
Conclusion

1. Presented and analyzed a model of learning about match quality with rehiring.

2. Found the optimal contract and characterized the value of experimentation.

3. Showed how to estimate easily structural models with Bayesian learning.