Minimum Distance Estimation of Heterogeneous Income Profile Model with Fixed Effects

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Econometrics Model of Earnings Dynamics

- The log income is often modeled:

$$y_{it} = \pi' \mathbf{x}_{it} + g_t + f_i + p_{it} + e_{it}$$

where

$$\begin{aligned} p_{it} &= \rho p_{it-1} + \eta_{it} \\ e_{it} &= \varepsilon_{it} + \mu \varepsilon_{it-1} \end{aligned}$$

to estimate the persistence of income shocks, and the variations of shocks (i.e. parameters of interests: $\rho, \mu, \sigma_\eta^2, \sigma_\varepsilon^2$)

- The **Minimum Distance (MD) estimation** minimizes the distance between the empirical and theoretical covariances of the unobserved components, $f_i + p_{it} + e_{it}$.

Heterogeneous Income Profiles (HIP)

- A HIP model assumes modest and less persistent income shocks and individuals having their own specific income profile. Then, the true log income is:

$$y_{it} = \pi' \mathbf{x}_{it} + g_t + f(h_{it}; \beta_i) + p_{it} + e_{it}$$

- Individual specific life-cycle earnings, $f(h_{it}; \beta_i)$, are often assumed to be a polynomial function of experience (linear or quadratic in experience). For example, a linear specification (Guvenen, 2009) is:

$$f(h_{it}; \beta_i) = \beta_{i,0} + \beta_{i,1} h_{it}$$

HIP and MD estimation

- The log income is:

$$y_{it} = \pi' \mathbf{x}_{it} + g_t + \beta_{i,0} + \beta_{i,1} h_{it} + p_{it} + e_{it}$$

  The MD estimation minimizes the distance between the empirical and theoretical covariances of the unobserved component of income, $\beta_{i,0} + \beta_{i,1} h_{it} + p_{it} + e_{it}$.

- For this, the individual specific parameters $\beta_{i,0}$ and $\beta_{i,1}$ are often assumed to be random in the sense that these parameters ($\beta_{i,0}$ and $\beta_{i,1}$) are uncorrelated with:
  - observed individual characteristics ($\mathbf{x}_{it}$)
  - initial shocks (p_{i0} and e_{i0})

Heterogeneous Income Profiles (HIP) with Fixed Effects

- The log income is:

$$y_{it} = \pi' \mathbf{x}_{it} + g(h_{it}; \gamma_t) + f(h_{it}; \beta_i) + p_{it} + e_{it}$$

where

$$\begin{aligned} g(h_{it}; \gamma_t) &= \gamma_{t,0} + \gamma_{t,1} h_{it} + \gamma_{t,2} h_{it}^2 = \gamma_t' H_{it} \\ f(h_{it}; \beta_i) &= \beta_{i,0} + \beta_{i,1} h_{it} + \beta_{i,2} h_{it}^2 = \beta_i' H_{it} \\ p_{it} &= \rho p_{it-1} + \eta_{it} \\ e_{it} &= \varepsilon_{it} \end{aligned}$$

- Our study allows for individual specific parameters ($\beta_{i,0}$ $\beta_{i,1}$, and, $\beta_{i,2}$) to be arbitrarily correlated with observed individual characteristics ($\mathbf{x}_{it}$).
- Initial shocks are assumed to be zero, and the initial shocks, if exist, are captured in $\beta_{i,0}$ as fixed effects.

HIP with Fixed Effects

- Allowing that individuals have their own income growth rates

- Imposing no restriction on $\beta_i = (\beta_{i,0}\ \beta_{i,1}, \beta_{i,2})$

  – $\beta_{i,0}\ \beta_{i,1}$,and $\beta_{i,2}$ are treated as fixed, i.e. parameters to estimate

  – $\beta_{i,0}\ \beta_{i,1}$,and $\beta_{i,2}$ can be correlated with each other

  – $\beta_{i,0}\ \beta_{i,1}$,and $\beta_{i,2}$ can be arbitrarily correlated with other time-varying regressors

- Controlling individual specific both time-invariant and time-varying characteristics ($\beta_{i,0}$ and $\mathbf{x}_{it}$)

Why Do Fixed Effects Matter?

- Observed time-varying characteristics $\mathbf{x}_{it}$ correlated with β_i.

  – e.g. marital status, household size, and etc.

- Some life-time events that people may call as shock but ex-post observable to researchers. These events are likely to be correlated with β_i.

  – e.g. lay-off, job changes, health shocks, and etc.

  – allowing the HIP coefficients to be fixed has a certain advantage in that we can directly control particular events that cause income fluctuations, if observed.

  – controlling such events can allow us to investigate about *pure* shock and also suggest the contribution of particular events on the income fluctuations.

Estimation Strategy

- π can be consistently estimated by the fixed effects estimation after g is concentrated out.
- $\beta_{i,0}$ $\beta_{i,1}$,and $\beta_{i,2}$ are treated as fixed, i.e. parameters to estimate, and so individual-by-individual OLS estimations precede the MD estimation:

  (For each individual, omitting i)

$$\tilde{y}_t = \beta_0 + \beta_1 h_t + \beta_2 h_t^2 + u, \quad \text{for } t = 1, \cdots T$$

- Then, $f(h_{it}; \beta_i) = \beta_{i,0} + \beta_{i,1} h_{it} + \beta_{i,2} h_{it}^2$ are concentrated out. However, when T is small, it causes biases (i.e. incidental trend problem). The biases due to small T are considered in calculating population moments, as well as the biases due to random missing.

Estimation Methodology

- Equally Weighted Minimum Distance Estimation (EWMD)

$$\hat{\Gamma}^{*}(t,k,\tau) \rightarrow_{p} \Gamma\left(\theta = \left(\rho, \mu, \sigma_{\eta}^{2}, \sigma_{\varepsilon}^{2}\right), t, k, \tau\right)$$

- Sample covariance:

$$\hat{\Gamma}^{*}(t,k,\tau) = \frac{1}{N_{\tau}^{*}} \sum_{i \in \mathcal{I}(\tau)} \hat{\tilde{\xi}}_{it}^{*} \hat{\tilde{\xi}}_{it-k}^{*}$$

- Theoretical covariance:

$$\Gamma(\theta,t,k,\tau) = \mathbb{E}\left(\hat{\tilde{\xi}}_{it}^{*} \hat{\tilde{\xi}}_{it-k}^{*}\right)$$

where

$$\hat{\tilde{\xi}}_{it}^{*} = \tilde{\xi}_{it}^{*} - H_{it}^{*\prime}\left(\sum_{t \in \mathcal{T}(\tau)} H_{it}^{*} H_{it}^{*\prime}\right)^{-1} \sum_{s \in \mathcal{T}(\tau)} H_{is}^{*} \tilde{\xi}_{is}^{*}$$

Model for Latent Income Process

- The true income process is assumed:

$$
\begin{aligned}
y_{it}^* &= \pi' \mathbf{x}_{it}^* + g(h_{it}^*; \gamma_t) + f(h_{it}^*; \beta_i) + \xi_{it}^* \\
&= \pi' \mathbf{x}_{it}^* + \gamma_t' H_{it}^* \qquad + \beta_i' H_{it}^* \qquad + p_{it}^* + \varepsilon_{it}^*
\end{aligned}
$$

where

$$p_{it}^* = \rho p_{it-1}^* + \eta_{it}^*$$

- Observed data and unobserved components:

$$
\begin{aligned}
y_{it} &= y_{it}^* s_{it}, \ \mathbf{x}_{it} = \mathbf{x}_{it}^* s_{it}, \ H_{it} = H_{it}^* s_{it} \\
\xi_{it} &= \xi_{it}^* s_{it}, p_{it} = p_{it}^* s_{it}, \ \varepsilon_{it} = \varepsilon_{it}^* s_{it}, \ \eta_{it} = \eta_{it}^* s_{it}
\end{aligned}
$$

where s_{it} is the dummy for the selection of the income process.

Model for Observed Income Process

- The observed income process is:

$$\begin{aligned} y_{it} &= \pi' \mathbf{x}_{it} + g(h_{it}; \gamma_t) + f(h_{it}; \beta_i) + \xi_{it} \\ &= \pi' \mathbf{x}_{it} + \gamma_t' H_{it} \qquad + \beta_i' H_{it} \qquad + p_{it} + \varepsilon_{it} \end{aligned}$$

where

$$p_{it} = \rho p_{it-1} + \eta_{it}$$

- Observed data and unobserved components:

$$\begin{aligned} y_{it} &= y_{it}^* s_{it},\ \mathbf{x}_{it} = \mathbf{x}_{it}^* s_{it},\ H_{it} = H_{it}^* s_{it} \\ \xi_{it} &= \xi_{it}^* s_{it}, p_{it} = p_{it}^* s_{it},\ \varepsilon_{it} = \varepsilon_{it}^* s_{it},\ \eta_{it} = \eta_{it}^* s_{it} \end{aligned}$$

where s_{it} is the dummy for the selection of the income process.

Empirical Population Moments

- For unbalanced data (with the assumption of random missing),

$$\hat{\tilde{\xi}}_{it} = \hat{\tilde{\xi}}^*_{it} s_{it} + o_p(1)$$

- Sample covariance:

$$\hat{\Gamma}(t, k, \tau) = \frac{1}{N_\tau} \sum_{i \in \mathcal{I}(\tau)} \hat{\tilde{\xi}}_{it} \hat{\tilde{\xi}}_{it-k}$$

- Its probability limit depends on the distribution of the missing data. Instead of using $\Gamma(\theta, t, k, \tau)$ as the probability limit, our study uses an estimate of $\Gamma(\theta, t, k, \tau)$, $\hat{\Gamma}(\theta, t, k, \tau)$, considering missing observation probabilities:

$$\hat{\Gamma}(\theta, t, k, \tau) = \mathbb{E}[\mathbb{E}[\hat{\tilde{\xi}}_{it} \hat{\tilde{\xi}}_{it-k} | \textit{non-missing}]]$$

Labor Income Data for Estimation and Simulation

- PSID, 1968-1996 (T=29)
  - 1,178 individuals with the restrictions: male head of household, 20-64 years old, positive hours and labor income, hourly labor earnings more than $2 and less than $400 in 1993, worked more than 520 hours and less than 5,110 hours, no SEO sample, at least 20 years residuals
  - 27 cohorts by grouping individuals who have experience level h-2 to $h+2$ and assigning the mid-point as the experience of that group
- Monte Carlo Simulation: unbalanced panel data replicating the PSID sample, in terms of the number of person-year observations and the patterns of missing

Monte Carlo Simulation Without and With X

	Without X			With X			
	ρ	σ_η^2	σ_ε^2	ρ	σ_η^2	σ_ε^2	π
True	0.750	0.050	0.050	0.750	0.050	0.050	0.500
Bias	0.009	-0.002	-0.003	0.002	0003	-0.004	0.000
Std	0.012	0.001	0.001	0.014	0.001	0.001	0.001
RMSE	0.088	0.012	0.012	0.077	0.013	0.012	0.001
True	0.850	0.050	0.050	0.850	0.050	0.050	0.500
Bias	0.020	-0.002	-0.003	-0.015	-0.002	-0.004	0.000
Std	0.012	0.001	0.001	0.065	0.001	0.002	0.001
RMSE	0.090	0.012	0.012	0.087	0.011	0.012	0.001

Estimation Result without Control Variables

	HIP with Fixed Effects			HIP with Random Effects		
	ρ	σ_η^2	σ_ε^2	ρ	σ_η^2	σ_ε^2
A	0.824	0.038	0.046	0.821	0.029	0.047
	(0.018)	(0.001)	(0.001)	(0.030)	(0.008)	(0.007)
C	0.798	0.043	0.042	0.805	0.025	0.032
H	0.831	0.037	0.048	0.829	0.022	0.034

The restuls for HIP with random effects are taken from Guvenen (2009).

Controlling Health Condition for an Application

- Health condition can be interpreted as individual's observed time-varying characteristics or as a source of income shocks.

- By controlling health conditions,

  – ρ, σ_η^2, and σ_ε^2 are the persistence and the variations of income shocks but only remaining part of income shocks excluding the health shock

  – it allows us to analyze the contribution of the health shock on the persistence and the variations of entire income shocks

Controlling Disability for an Application

- Self-reported disability condition from the question in PSID:

  "Do you have a physical or nervous limitation that limits the amount or type of work you can do?"

$$x_{it} = \begin{cases} 1 & \text{if yes} \\ 0 & \text{if no} \end{cases}$$

- The validity of the measurement is somewhat controversial (mainly due to subjectivity, justification for non-participation in labor force, endogeneity). While not perfect, offer the best available method of measurement (Meyer and Mok, 2009).
- Why do fixed effects matter?

  One's disability can be correlated with their own specific parameters (both individual time-invariant characteristics and growth parameters).

Basic Statistics for Disability Data

A. Disability Rates (pooled years)

	A	C	H
<35	5.6	4.6	6.0
35-55	8.2	6.9	8.7
>55	13.0	13.1	12.9
All	7.6	6.6	8.1

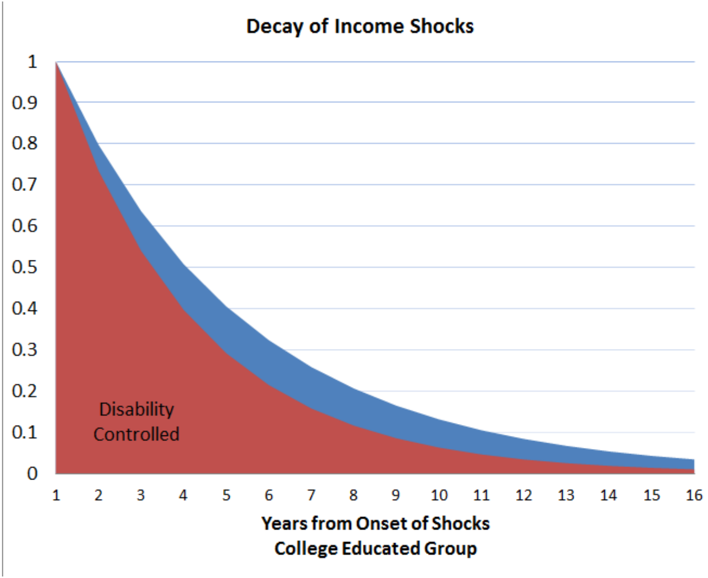
B. Disability Prevalence (across individuals)

	A	C	H
At least once	39.7	33.7	42.4
One to three times	23.3	20.2	24.8
More than four times	16.4	13.5	17.6
More than four times for 10 years	14.1	10.8	15.6

HIP Estimation Result Controlling Disability

	Without X			With X			
	ρ	σ_η^2	σ_ε^2	ρ	σ_η^2	σ_ε^2	π
A	0.824	0.038	0.046	0.782	0.039	0.044	-0.057
	(0.018)	(0.001)	(0.001)	(0.018)	(0.001)	(0.001)	(0.008)
C	0.798	0.043	0.042	0.735	0.043	0.037	-0.051
H	0.831	0.037	0.048	0.803	0.037	0.047	-0.059

Persistence of Income Shock Controlling Disability



Conclusion

- This study proposes a way to estimate a fixed effect heterogeneous income profile model where the parameters of the HIP are treated as fixed
- Our simulation suggests that our estimator works well especially for T=29 and ρ is around 0.75 and 0.85
- As an application, we consider disability status as a control variable. Under certain assumptions, our estimation results indicate that disability shock somewhat accounts for the persistence of the income shock.

Discussion

- **[Correlated random effect approach]** Browning et al (2010) allow that the HIP coefficients are correlated with initial income in the income dynamics equation. To estimate the model, they use the simulated method of moments with a full parametric specification of the correlated random effects.
- Compared to the correlated random effect approach, our approach do not specify the parametric relationship between the HIP parameters and the explanatory variables.
- Our study allows that the HIP trend coefficients are correlated with time-constant unobserved characteristics that contribute to the initial income, but initial income beyond the time-constant unobserved characteristics (this may be called initial shock) is assume to be zero.