Understanding Earnings Dynamics: Identifying and Estimating the Changing Roles of Unobserved Ability, Permanent and Transitory Shocks

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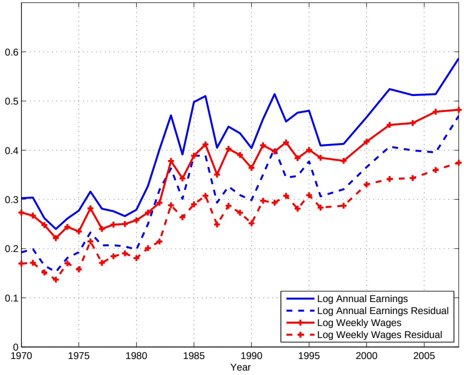
Role of Unobserved Ability/Skills

- There is considerable interest in the evolution of inequality and the returns to ability/skill over time
- Widespread agreement that returns to observed skills (education, experience) have risen since the early 1980s
- Less agreement on role of unobserved skills
  - Autor, Katz and Kearney (2008) vs. Card & DiNardo (2002), Lemieux (2006)
- More generally, there is interest in understanding the factors driving the evolution of residual inequality

Earnings and Weekly Wage Inequality in the US

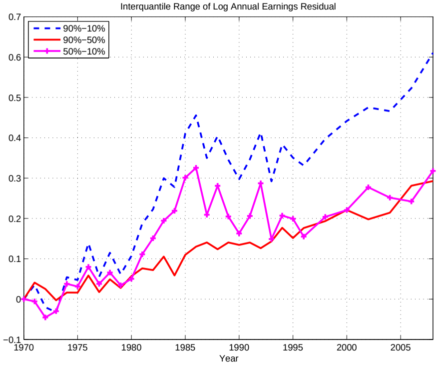


Source: 1970-2008 PSID

Residuals and Unobserved Ability/Skill

- CPS-based literature interprets all changes in residual inequality as changes in the 'pricing' of unobserved skills
  - e.g., Katz & Murphy (1992), Juhn, Murphy & Pierce (1993), Autor, Katz & Kearney (2008)
  - increased residual inequality reflects an increase in the 'returns' to unobserved skill
- Along with increase in returns to observed skill, this literature has motivated theories of SBTC (e.g. Acemoglu 1999, Caselli 1999, Galor & Moav 2000, Violante 2002)
- Changes in institutional factors and minimum wages may also be important (Card & DiNardo 2002, Lemieux 2006)
- Most recently, Acemoglu & Autor (2011) and Autor & Dorn (2012) argue that mechanization of routine tasks has led to polarization of skill demand

Interquantile Comparisons for Log Earnings Residuals, 1970-2008 PSID



What about Idiosyncratic Shocks?

- CPS-based literature largely ignores parallel research on earnings dynamics using PSID
  - labor: Gottschalk & Moffitt (1994, 2002, 2009, 2012), Haider (2001), Meghir & Pistaferri (2004), Robin & Bonhomme (2010)
  - macro: Heathcote, Perri & Violante (2010), Heathcote, Storesletten & Violante (2010)
- Decomposes variance of log wages/earnings residuals into permanent and transitory shocks over time
- Important for understanding consumption and savings behavior/inequality
- Estimates suggest similar increases in variance of both permanent and transitory shocks
- Transitory component unlikely to be related to unobs. skill
- Rarely account for changes in pricing of unobs. skills

Our Goal: Incorporating All Three Components

We consider a general log earnings/wage residual decomposition:

$$
\begin{aligned}
W_{it} &= \mu_t(\theta_i) + \kappa_{i,t} + \nu_{i,t} \\
\kappa_{i,t} &= \kappa_{i,t-1} + \eta_{i,t} \\
\nu_{i,t} &= \xi_{i,t} + \beta_{1t}\xi_{i,t-1} + \beta_{2t}\xi_{i,t-2} + \ldots + \beta_{qt}\xi_{i,t-q}
\end{aligned}
$$

- 'Unobserved Ability' literature effectively ignores any changes in distributions of $\kappa_{i,t}$ and $\nu_{i,t}$
- 'Earnings Dynamics' literature effectively ignores $\mu_t(\theta_i)$ or assumes $\mu_t(\theta_i)$ is time invariant

Earnings Components

$$
\begin{aligned}
W_{it} &= \mu_t(\theta_i) + \kappa_{i,t} + \nu_{i,t} \\
\kappa_{i,t} &= \kappa_{i,t-1} + \eta_{i,t} \\
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\end{aligned}
$$

- $\mu_t(\cdot)$ reflects the pricing of unobserved skills θ, which may change over time due to technological change or institutional factors (e.g. unions, minimum wage)
- η_t reflects permanent idiosyncratic shocks like job displacement, switching employers, disability
- ν_t reflects potentially persistent but transitory shocks like temporary illness, family disruption, temporary demand shocks for employers

Changing Skill Prices vs. Shocks

- Changing skill prices affect earnings of similar individuals in the same way – strong co-movements over time
- Idiosyncratic shocks can move the wages/earnings of similar workers in very different directions
- We think of skill pricing functions as relatively slow moving based on supply/demand and institutional factors (e.g. unions, minimum wages)
  - likely to be more predictable
- Predictable changes in skill prices should have weak effects on within-cohort consumption inequality but should increase inequality across cohorts
- Increased variance of permanent shocks should increase within- and across-cohort consumption inequality
- Skill prices and variability of shocks have different implications for precautionary savings

What We Do – Outline

- We first consider conditions required for nonparametric identification
- Consider a moment-based approach for estimation
  - briefly discuss necessary conditions for identification with polynomial $\mu_t(\cdot)$ functions
  - provide Minimum Distance estimates assuming $\mu_t(\cdot)$ are linear/cubic polynomials
  - focus on log earnings residuals for men in PSID, 1970-2008

Nonparametric Identification: Simple Case

Consider non-parametric identification, beginning with a simple instructive case:

$$W_{it} = \mu_t(\theta_i) + \varepsilon_{it}$$

- ε_{it} are independent over time
- $\mu_t(\cdot)$ are strictly increasing

Some normalizations:

- $E(\theta) = E(\varepsilon_t) = 0$
- $\mu_1(\theta) = \theta$

Problem is very similar to that of measurement error literature (e.g. Hu and Schennach 2008)

Some Intuition on Identifying $\mu_t(\theta)$ and $f_\theta(\cdot)$

- If $\mu_t(\theta) = m_{0,t} + m_{1,t}\theta$, then the problem is just like a standard measurement error problem with multiple measurements
- If $W_1 = \theta$, we could just regress W_t on W_1 to identify $\mu_t(\cdot)$
- Due to ε_1, we would get attenuation bias
- Can use other $W_{t'}$ as instruments in a regression of W_t on W_1 to identify $\mu_t(\cdot)$
- Can then identify σ_θ^2
- General case is a bit like nonparametric IV in context of measurement error

Assumption 1

The following conditions hold for $T = 3$:

(i) The joint density of θ, W_1, W_2, and W_3 is bounded and continuous, and so are all their marginal and conditional densities.

(ii) W_1, W_2, and W_3 are mutually independent conditional on θ.

(iii) $f_{W_1|W_2}(w_1|w_2)$ and $f_{\theta|W_1}(\theta|w_1)$ form a bounded complete family of distributions indexed by W_2 and W_1, respectively.

▸ Definition

(iv) For all $\bar{\theta}, \tilde{\theta} \in \Theta$, the set $\{w_3 : f_{W_3|\theta}(w_3|\bar{\theta}) \neq f_{W_3|\theta}(w_3|\tilde{\theta})\}$ has positive probability whenever $\bar{\theta} \neq \tilde{\theta}$.

(v) We normalize $\mu_1(\theta) = \theta$ and $E[\varepsilon_t|\theta] = 0$ for all t.

Lemma 1: Identification in the Simple Case

Lemma 1

Under Assumption 1, $f_\theta(\cdot)$, $f_{\varepsilon_t}(\cdot)$, and $\mu_t(\cdot)$ are identified $\forall t$.

Proof:

- Thm 1 of Hu and Schennach (2008) gives identification of $f_{W_1|\theta}(\cdot|\cdot)$, $f_{W_2|\theta}(\cdot|\cdot)$, and $f_{W_3,\theta}(\cdot,\cdot)$ from $f_{W_1,W_2,W_3}(\cdot,\cdot,\cdot)$
- $f_\theta(\cdot)$ can be recovered from $f_{W_3,\theta}(\cdot,\cdot)$ by integrating out W_3 (Cunha, Heckman and Schennach 2010)
- identify $\mu_2(\cdot)$ and $\mu_3(\cdot)$ from $E[W_t|\theta] = \mu_t(\theta)$ given $f_{W_t|\theta}(\cdot|\cdot)$
- $f_{\varepsilon_t}(\cdot)$ is identified from $f_{\varepsilon_t}(\varepsilon) = f_{W_t|\theta}(\mu_t(\theta) + \varepsilon)$, since $\mu_t(\cdot)$ and $f_{W_t|\theta}(\cdot|\cdot)$ are already known.

Nonparametric Identification: Serially Correlated Shocks

Now, consider heteroskedastic permanent shocks and an $MA(1)$ process for ε_{it}:

$$\begin{aligned} W_{it} &= \mu_t(\theta_i) + \kappa_{i,t} + \nu_{i,t} \\ \kappa_{i,t} &= \kappa_{i,t-1} + \eta_{i,t} \\ \nu_{i,t} &= \xi_{i,t} + \beta_t \xi_{i,t-1} \end{aligned}$$

- Allow $\eta_{i,t} = \sigma_t(\theta_i)\zeta_{i,t}$
- Identification for most parameters/densities/functions requires $T \geq 9$ (we focus on $T = 9$)
- Use differences to eliminate correlations in shocks

Generalizing Lemma 1

Taking first differences and looking at observations far enough apart, we can get back to independence and apply Lemma 1 (or something similar):

$$
\begin{aligned}
W_1 &= \theta + \varepsilon_1 = \theta + \{\eta_1 + \nu_1\} \\
\Delta W_4 &= \Delta\mu_4(\theta) + \Delta\varepsilon_4 = \Delta\mu_4(\theta) + \{\eta_4 + \Delta\nu_4\} \\
\Delta W_7 &= \Delta\mu_7(\theta) + \Delta\varepsilon_7 = \Delta\mu_7(\theta) + \{\eta_7 + \Delta\nu_7\}
\end{aligned}
$$

- repeat for other triplets $(W_2, \Delta W_5, \Delta W_8)$ and $(W_3, \Delta W_6, \Delta W_9)$ to identify all $\mu_t(\cdot)$
- identification now comes from relationship between earnings and future earnings changes

Assumption 2

The following conditions hold for $T = 9$:

(i) The joint density of θ, W_1, W_2, W_3, $\Delta W_4, \ldots, \Delta W_9$ is bounded and continuous, and so are all their marginal and conditional densities. $f_\theta(\cdot)$ is non-vanishing on $\mathbb{R}$.

(ii) Unobserved components ζ_t, ξ_t, and θ are mutually independent for all $t = 1, \ldots, 9$.

(iii) $f_{W_t|\Delta W_{t+3}}(w_t|\Delta w_{t+3})$ and $f_{\theta|W_t}(\theta|w_t)$ form a bounded complete family of distributions indexed by ΔW_{t+3} and W_t, respectively, for $t = 1, 2, 3$.

(iv) For all $\bar{\theta}, \tilde{\theta} \in \Theta$ and $t = 7, 8, 9$, the set $\{w : f_{\Delta W_t|\theta}(w|\bar{\theta}) \neq f_{\Delta W_t|\theta}(w|\tilde{\theta})\}$ has positive probability whenever $\bar{\theta} \neq \tilde{\theta}$.

(v) We impose the following normalizations: $\kappa_0 = \xi_0 = 0$, $\mu_1(\theta) = \theta$, $E[\zeta_t] = E[\xi_t] = 0$, $E[\zeta_t^2] = 1$, and $\sigma_t(\cdot) > 0$ for all t.

(vi) For all t, we assume the Carleman's condition holds for ζ_t and ξ_t.

Theorem 1: Identification with Serially Correlated Errors

Theorem 1

Under Assumption 2, $f_\theta(\cdot), \{f_{\eta_t}(\cdot), f_{\xi_t}(\cdot), \beta_t\}_{t=1}^{7}$, *and* $\{\mu_t(\cdot)\}_{t=1}^{9}$ *are identified.*

Proof has three steps:

- Identify $f_\theta(\cdot)$ and $\mu_t(\cdot)$ for all t using Lemma 1. Details
- Identify $E[\xi_t^2]$, β_t, and $\sigma_t(\cdot)$ for $t = 1, ..., 7$ using various second moments.
- Identify $f_{\eta_t}(\cdot)$ and $f_{\xi_t}(\cdot)$ for $t = 1, ..., 7$.

Some General Comments on Identification

- For $T \geq 9$, this general strategy can be used to identify $f_\theta(\cdot)$, $\{\mu_t(\cdot)\}_{t=1}^{T}$ and $\{f_{\eta_t}(\cdot), f_{\xi_t}(\cdot), \beta_t\}_{t=1}^{T-2}$
- If shocks are all homoskedastic, cannot have flat regions in $\Delta\mu_t(\cdot)$ for $t = 7, 8, 9$
- Identification approach rules out an autoregressive process where transitory shocks never die out
- Can handle arbitrarily long $MA(q)$ process, but may require a long panel
  - $MA(q)$ requires $T \geq 6 + 3q$ time periods
  - can only identify shock process through $T - q - 1$

Moment-Based Approach

Now, consider a moment-based approach

- Assume $\mu_t(\theta) = m_{0,t} + m_{1,t}\theta + ... + m_{p,t}\theta^p$
- Normalize $\mu_1(\theta) = \theta$ and $E[\mu_t(\theta)] = 0$ for $t = 2, ..., T$
- Assume $f_{\xi_t}(\cdot)$ and $f_{\eta_t}(\cdot)$ are time-specific
- Assume θ, η_t and ξ_t are mutually independent with η_t and ξ_t independent over time
- Normalize $E[\theta] = E[\eta_t] = E[\xi_t] = 0$

Using Variances & Covariances

Assuming shocks begin at age $a = 1$, we have variances:

$$E[W^2_{i,a,t}|a,t] = \sum_{j=0}^{p}\sum_{j'=0}^{p} m_{j,t}m_{j',t}E[\theta^{j+j'}] + \sum_{j=0}^{a-1}\sigma^2_{\eta_{t-j}} + \sigma^2_{\xi_t} + \sum_{j=1}^{\min\{q,a-1\}} \beta^2_{j,t}\sigma^2_{\xi_{t-j}}$$

and covariances (for $l \geq 1$):

$$E[W_{i,a,t}W_{i,a+l,t+l}|a,t,l] = \sum_{j=0}^{p}\sum_{j'=0}^{p} m_{j,t}m_{j',t+l}E[\theta^{j+j'}] + \sum_{j=0}^{a-1}\sigma^2_{\eta_{t-j}} + E(\nu_{i,a,t}\nu_{i,a+l,t+l})$$

Number of Moments and Parameters for $MA(1)$

Consider the number of moments & parameters for one cohort using variances & covariances

Number of parameters:

- $2p - 1$ parameters for $E[\theta^2]$,...,$E[\theta^{2p-1}]$,$E[\theta^{2p}]$
- $(p + 1)(T - 1)$ parameters for $\mu_t(\theta)$ polynomials, $t = 2, ..., T$
- $2(T - 2)$ parameters for $\sigma^2_{\eta_t}$ and $\sigma^2_{\xi_t}$, $t = 1, ..., T - 2$
- $T - 3$ parameters for β_t, $t = 2, ..., T - 2$
- Total number of parameters: $(4 + p)T + p - 9$

Number of moments:

- $\frac{T(T+1)}{2}$ variance/covariance terms
- $T - 1$ moments coming from $E[\mu_t(\theta)] = 0$, $t = 2, ..., T$
- Total moments: $\frac{T(T+1)}{2} + T - 1$

Identification

Necessary condition for identification: $T \geq 3$ and $p \leq \frac{T^2 - 5T + 16}{2(T+1)}$

- Cubic $\mu_t(\cdot)$ requires $T \geq 10$
- Adding higher residual moments can be helpful
- Higher moments necessary to identify higher moments of shock distributions $f_{\eta_t}(\cdot)$ and $f_{\xi_t}(\cdot)$

Multiple Cohorts

- With changing distribution of cohorts over time (aging in and out of panel), it is important to account for the fact that older cohorts have accumulated a longer history of shocks
  - we assume shocks start at age 20
- Additional cohorts can aid in identification, since $\mu_t(\cdot)$ does not vary across cohorts

PSID Data: Overview

- PSID is a longitudinal survey of a representative sample of US individuals and their families
- Collected annually through 1997, biennially starting in 1999
- We use data from interview years 1971 through 2009
- Earnings are collected for the previous year, so data cover calendar years 1970-2008
- Earnings: household head's total wages and salaries (excluding farm and business income)
- Earnings reported in 1996 dollars using CPI-U-RS

Sample Restrictions

- Core (SRC) sample with nonzero weights
  - exclude oversamples (SEO, Latino) and nonsample persons
- Male heads of households
- Ages 30-59
- Positive annual wages and weeks worked
- Non-students
- Trim top and bottom 1% of wages within each age-year cells (ten-year age group used)
- Resulting data set has 3,302 men and 33,207 person-year observations

Sample Statistics

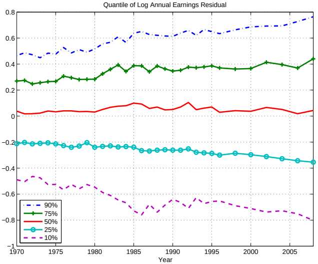
- Race: 92% White, 6% black, 1% hispanic
- Age: mean age is 47
- Educational Attainment

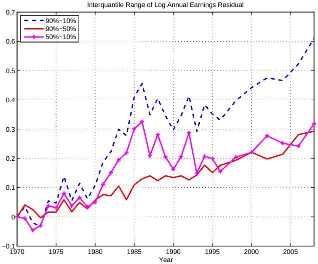
Education (years)	Percent
Elementary (1-5)	1.2
Middle (6-8)	5.0
Some High (9-11)	9.9
Completed High (12)	33.7
Some College (13-15)	20.0
Completed College (16)	20.6
Advanced Degrees (17+)	9.8

Obtaining Residuals

- We focus on the distribution of residual earnings, controlling for differences in education, race, and age
- Run a cross-sectional regression of log earnings for each year on
  - age dummies
  - race dummies
  - education dummies
  - race dummies $\times$ cubic polynomial in age
  - education dummies $\times$ cubic polynomial in age

Residual Earnings Inequality in the US, 1970-2008





Moment-Based Estimation

- Assume $\beta_{j,t} = \beta_j$ for all j and t
- We assume $\sigma^2_{\eta_\tau} = \sigma^2_{\eta_0}$ and $\sigma^2_{\xi_\tau} = \sigma^2_{\xi_0}$ for all τ years prior to our data
  - other assumptions yield similar results
- Assume homoskedastic shocks for most of the analysis
- Use minimum distance for estimation
  - aggregate into three age categories for variance/covariance moments
  - weight moments by share of observations used for that moment

Decomposition

We focus on decomposing variance of earnings residuals into three components:

- pricing of unobserved skills: $Var[\mu_t(\theta)]$
- permanent shocks: $\sigma^2_{\kappa_t} = \sum_{j=0}^{a-1} \sigma^2_{\eta_{t-j}}$
- transitory shocks: $\sigma^2_{\nu_t} = \sigma^2_{\xi_t} + \sum_{j=1}^{q} \beta^2_{jt} \sigma^2_{\xi_{t-j}}$

Estimation with linear $\mu_t(\theta)$

We begin by assuming $\mu_t(\theta)$ is linear

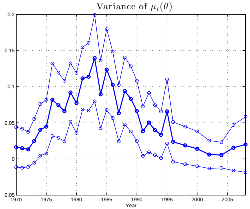
- only use variances & covariances in estimation
- begin by assuming distribution of θ is the same across cohorts, but explore changes across cohorts later

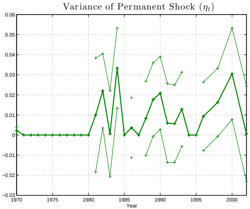
MD estimation

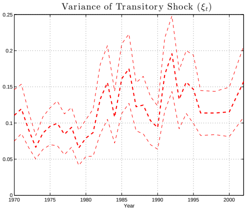
	Constant μ_t	Time-Varying $\mu_t(\cdot)$			
	$MA(3)$	$MA(1)$	$MA(2)$	$MA(3)$	$MA(5)$
Min. Obj. Fun.	**168.27**	130.73	124.16	**121.10**	116.93
β_1	0.326	0.297	0.281	0.288	0.299
	(0.027)	(0.033)	(0.026)	(0.027)	(0.027)
β_2	0.222	·	0.186	0.172	0.194
	(0.025)	·	(0.025)	(0.021)	(0.020)
β_3	0.246	·	·	0.141	0.137
	(0.034)	·	·	(0.025)	(0.022)
β_4	·	·	·	·	0.126
	·	·	·	·	(0.020)
β_5	·	·	·	·	0.084
	·	·	·	·	(0.024)

Variance of θ and shocks

MA(3) model with time-varying $\mu_t(\theta)$

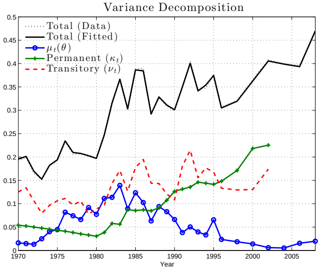


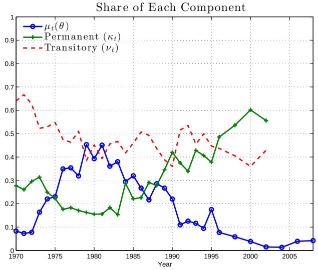




Variance Decomposition

MA(3) model with time-varying $\mu_t(\theta)$

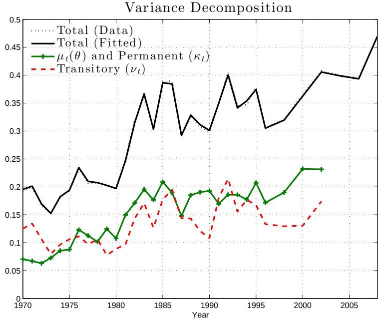


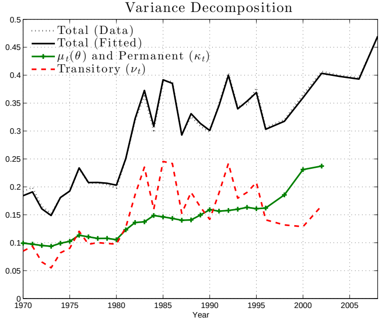


Comparison with time-invariant μ model

MA(3) shocks with time-varying vs. time-invariant $\mu_t(\theta)$

Time-Varying $\mu_t(\theta)$	Time-Invariant $\mu(\theta)$

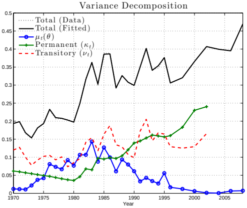




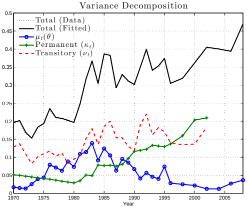
Other Transitory Processes

MA(1), MA(5), and ARMA(1,1) shocks with time-varying $\mu_t(\theta)$

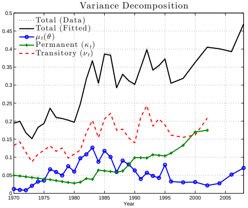
MA(1)



MA(5)



ARMA(1,1)



η_t as Skill Shocks

Suppose we interpret η_t as skill shocks so

$$W_{it} = \mu_t(\theta_i + \kappa_{it}) + \nu_{it}$$

where we assume linear $\mu_t(\cdot)$

η_t as Skill Shocks

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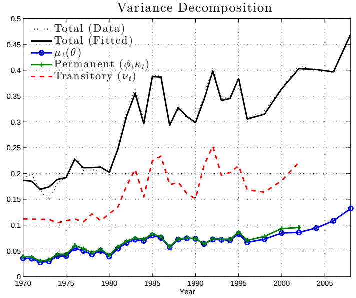
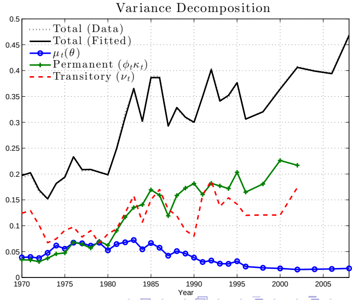
where we assume linear $\mu_t(\cdot)$

Constant $\sigma^2_{\eta t}$



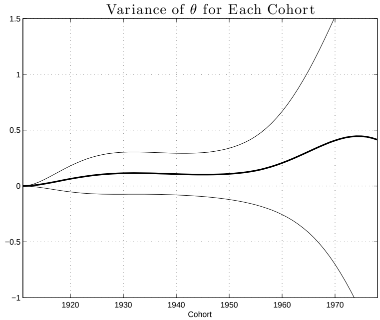
Time-Varying $\sigma^2_{\eta t}$

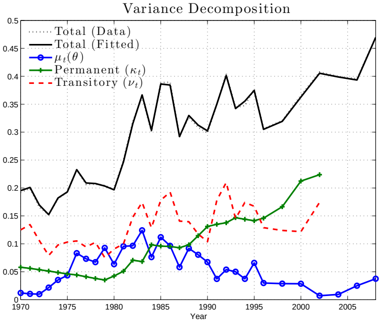


Allowing for cohort differences in distribution of θ

- Shifts in mean of θ are absorbed in age and time effects before obtaining residuals
- We examine whether the variance of θ varies across cohorts
  - birth cohorts from 1911 to 1978
  - assume a cubic spline in year of birth with two interior knots

Cohort Differences in Variance of θ





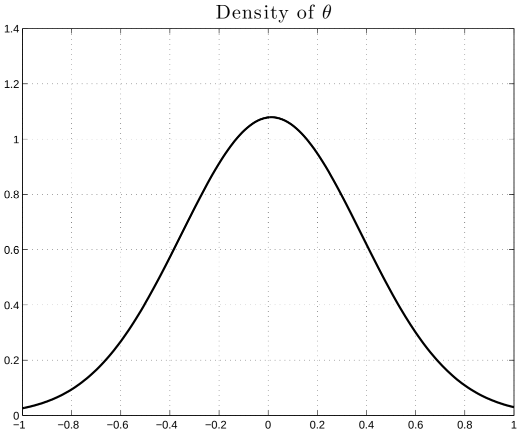
Estimation with cubic $\mu_t(\theta)$

Now, assume

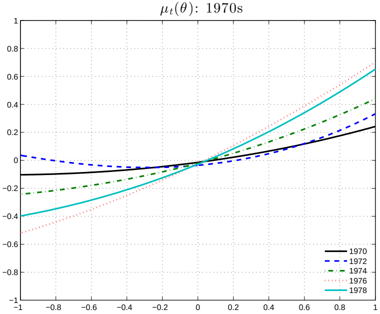
- $\mu_t(\theta)$ are time-varying cubic functions with $\mu_{1985}(\theta) = \theta$
- $f_\theta(\cdot)$ is a mixture of two normals (same for all cohorts)
- permanent and MA(3) transitory shocks

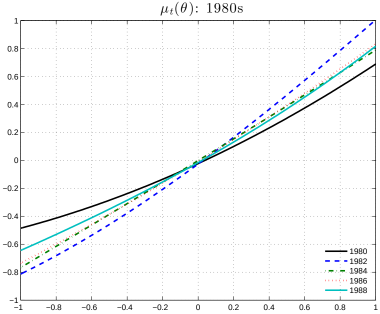
We now use all second- and third-order moments of log earnings residuals

Distribution of θ (1985)

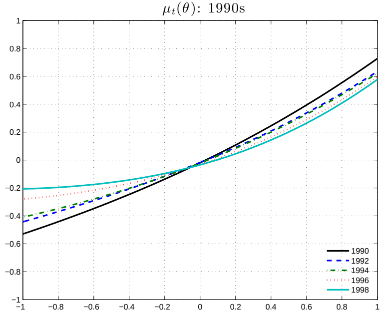


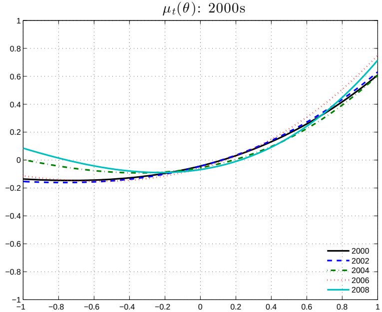
Estimated $\mu_t(\theta)$ functions





Estimated $\mu_t(\theta)$ functions

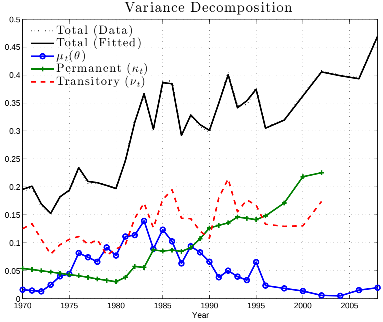




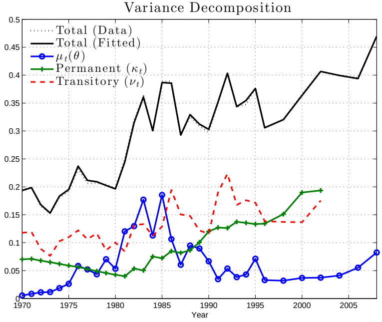
Variance Decomposition

Comparison with linear $\mu_t(\theta)$

linear $\mu_t(\theta)$

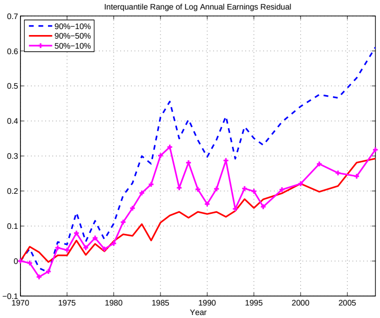


cubic $\mu_t(\theta)$

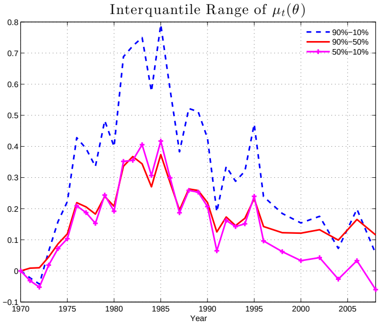


Evolution of $\mu_t(\theta)$ distribution vs. residual distribution

Residuals

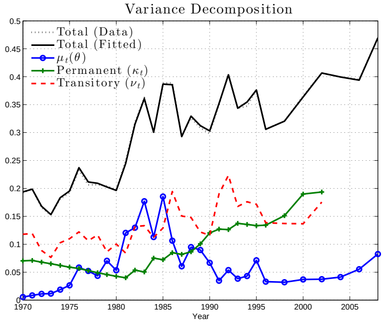


$\mu_t(\theta)$

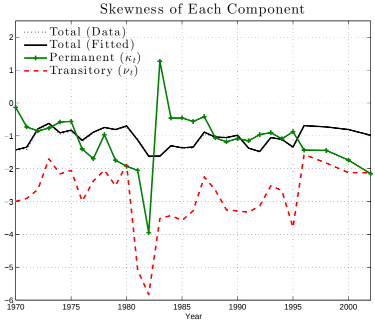


Variances and Skewness Over Time

Variance

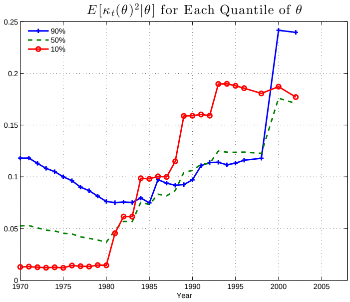


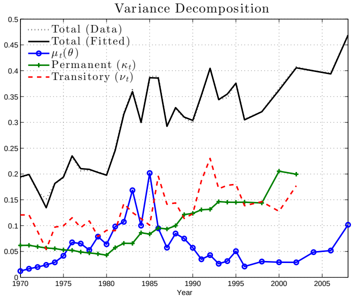
Skewness



Heteroskedasticity in Permanent Shocks

Consider $\sigma_t(\theta) = s_{0,t} + s_{1,t}\theta$.





Summary & Conclusions

- We consider identification and estimation of a model with unobserved skill differences and time-varying
  - skill 'pricing' functions
  - permanent shocks
  - transitory shocks
- Identification
  - prove nonparametric identification
  - discuss identification for a moment-based approach
- Estimation
  - Minimum Distance estimation using second- and third-order residual moments
  - Use male log earnings residuals in PSID

Summary & Conclusions

- Results suggest that all components of earnings have played an important role since 1970
  - ‘returns’ to unobserved skill increased broadly in 1970s and early 1980s but fell in late 1980s/early 1990s
    - stronger decline in value of unobserved skill at bottom than top after 1995 (partial polarization)
  - variance of unobserved skills changed little over 1925-55 cohorts
  - variance of transitory shocks jumped in early 1980s and bounced around after
  - variance of permanent shocks rose consistently over 1980s and 1990s (especially among low ability)
- Inequality in unobserved skills evolves quite differently from overall residual inequality
  - accounting for idiosyncratic shocks is important for understanding role of unobserved skills

An Economic Explanation

A theory based on slow diffusion of skill-biased technology in frictional labor markets can be helpful (based on Violante 2002)

- introduction of skill-biased technology in 1970s
- quick diffusion to most high-skill workers
  - increase in $\mu_t'(\cdot)$ but not $\sigma^2_{\eta_t}$
- by mid-1980s, skill begins to diffuse more slowly to lower skilled workers
  - decrease in $\mu_t'(\cdot)$
- matching of low-skill workers to new technologies random due to market frictions
  - increase in $\sigma^2_{\eta_t}(\theta)$ for low ability workers
- rising $\sigma^2_{\eta_t}$ consistent with rising variance of wages paid across firms (e.g., Dunne et al. 2004, Barth et al. 2011)

Future Efforts

- Estimate differences by education and/or race
  - allow $f_\theta(\cdot)$ and $\mu_t(\theta)$ to vary by education/race
  - what roles do changes in unobserved skill distributions and pricing play in earnings gaps?
- Move from modelling residuals to earnings/wages themselves
- Allow for multiple unobserved skills
- Examine implications for consumption inequality within and across cohorts over time

Thanks

Bounded complete family of distributions

$f_{\theta|W}(\theta|W)$ forms a bounded complete family of distributions indexed by W if $g(\theta) = 0$ is the only bounded function that solves:

$$\int g(\theta) f_{\theta|W}(\theta|W) d\theta = 0, \quad \forall W$$

- Standard assumption in nonparametric identification literature related to invertability of conditional expectation integral function
- E.g. violated if
  - $\theta \perp\!\!\!\perp W$, since $g(\theta) = \theta - E(\theta)$ solves the equation above
  - $f_{\theta|W}$ is symmetric about 0 for all W, e.g. W only affects variance of θ

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Step 1: Details on identification of μ_2, μ_3,...

Consider the second subset of equations:

$$\begin{aligned}
W_2 &= \mu_2(\theta) + \varepsilon_2 = \theta_2 + \{\eta_1 + \eta_2 + \nu_2\} \\
\Delta W_5 &= \Delta\mu_5(\theta) + \Delta\varepsilon_5 = g_5(\theta_2) + \{\eta_5 + \Delta\nu_5\} \\
\Delta W_8 &= \Delta\mu_8(\theta) + \Delta\varepsilon_8 = g_8(\theta_2) + \{\eta_8 + \Delta\nu_8\}
\end{aligned}$$

where $g_t(\theta_2)$ is implicitly defined by $\Delta\mu_t(\theta) = g_t(\mu_2(\theta))$.

- Can identify $f_{\theta_2}(\cdot)$, $g_5(\cdot)$, and $g_8(\cdot)$ using same approach
- Recover the function $\mu_2(\cdot)$ by $\mu_2(\theta) = F_{\theta_2}^{-1}(F_\theta(\theta))$
- Once we identify $\mu_2(\cdot)$, $\Delta\mu_5(\cdot)$ and $\Delta\mu_8(\cdot)$ are identified from $\Delta\mu_t(\theta) = g_t(\mu_2(\theta))$

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