Post Schooling Human Capital Investments and the Life Cycle Variance of Earnings

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Motivation: Life Cycle Variance of Earnings

Questions about life cycle earnings inequality:

- decomposition into permanent and transitory effects
- evaluation of explanatory power of macro drivers of inequality

Models of earning dynamics:

- Many reduced-form competing models (Meghir and Pistaferri, 2010 for a survey) like random growth or HIP vs random walk or RIP models that are difficult to discriminate (Baker, 1997).
- structural models of human capital investments based on Ben Porath (1967). Tightly specified models solved by backward induction (Browning, Hansen and Heckman, 1999, or Rubinstein and Weiss, 2006, for a survey). Identification is fragile and frequently, conditional on parametric specification and strong restrictions.on the dimension of heterogeneity.
  - Illustrate the second part of Mincer's research program on post schooling wage growth in which the *stylized facts* are:
    - Mean earnings grow at a rate which decreases with one's working lifetime.
    - Variances of earnings: first decreasing then increasing over working lifetime

This paper

- Builds a bridge between reduced & structural forms. Exhibit conditions under which a structure leads to a simple and identifiable reduced form
- Results in a linear factor model for log-earnings in which factor loadings are interpretable as structural **individual specific** terms like rates of returns, costs etc.
- Tests the structural restrictions involved in this construction.
- Simulates the counterfactual impact of changes in returns to investments or other important parameters (survival probabilities, retirement compulsory age,...) in macromodels but with lots of heterogeneity.

Literature

Earning models:

- *Human capital investments over the life cycle* : Ben Porath, 1967, Mincer, 1974, Haley, 1976, Heckman, Lochner and Todd, 2006, Rubinstein and Weiss, 2006, Polachek, Das and Thamma-Apiroam, 2013,.
- *Learning and Search*: Farber and Gibbons (1996), Postel Vinay and Turon (2010) among others
- *Mobility*: Buchinsky, Fougère, Kramarz and Tchernis, 2010, Altonji, Smith and Vidangos, 2009.

The two last ones are concerned by "frictions" in the labour market. The first one disregards frictions and assume that human capital is homogenous and priced by a single price up to idiosyncratic shocks.

Estimation of earning equations:

- *Dynamic panel data, covariance structures and other methods:* Lillard and Willis (1978), Hause (1980), MaCurdy (1982), Abowd and Card (1989), Baker (1997), Geweke and Keane (2000), Hirano (2002), Meghir and Pistaferri (2004), Guvenen (2009), Alvarez, Browning and Ejrnaes (2010), Hryshko (2012) among many others.
- *Factor models:* Cross section dependence: Pesaran (2006), Bai (2009), Moon and Weidner (2010).
- *Factor models in earnings*: Cunha, Heckman and Urzua (2007), Bonhomme and Robin (2009, 2010), Arellano and Bonhomme (2010).

Outline of the Talk

- *Human capital model*: inspired by albeit slightly different than Ben Porath (1967). Generating a factor model for log earnings with three unobserved factor loadings: levels, growth and curvature summarize earnings profiles (in the spirit of Lillard and Reville, 1999). Factors loadings are interpretable in terms of individual specific returns, costs to investments or value function parameters and are restricted by economic structure.
- *Data*: French social security records on a single (labor market) entry cohort observed over 30 years of around 7,500 observations. France is characterized by a very **stable** earnings inequality over these years.
- *Empirical strategy*: A decomposition between/within groups or market prices/individual specific investments and frictions, and a sequence of random effect and fixed effect methods.
- *Test of* structural restrictions on growth and curvature factor loadings.
- *Counterfactual analysis*: increasing terminal returns to investment (i.e. increased life-expectancy) increase cross-section means and variances in later years of the life-cycle.

Outline of results

- The longer the working period, the more high-return investors reap benefits from investing and income inequality increases
- Quite difficult to reject human capital investment model (with heterogeneous growth)
- The share of permanent factors is increasing with potential experience (from 3 to 88%, mean 64%)
- A crucial (reduced form) parameter is the stopping time of investments arising from our set-up that differs from Ben Porath (1967). This parameter is not identified from the data.

Theoretical set-up: Earnings and Human Capital Stocks

Timing:

- Entry in the labor market at $t = 1$, terminal date at $t = T$ (chosen arbitrarily)

Potential earnings:

$$y_i^P(t) = \exp(\delta_i(t))H_i(t)$$

- $\delta_i(t)$ is the (log)-price of a single dimensional human capital stock.
- $H_i(t)$ is the human capital stock.

Actual earnings net of investments in human capital:

$$y_i(t) = \exp(\delta_i(t))H_i(t)\exp(-\tau_i(t))$$

$1 - \exp(-\tau_i(t))$ can be interpreted as the fraction of working time devoted to investing in a single-dimensional human capital whose price is period-dependent (Ben Porath, 1967).

Accumulation:

$$H_i(t+1) = H_i(t)\exp[\rho_i\tau_i(t) - \lambda_i(t)]$$

- ρ_i is the rate of return to human capital
- $\lambda_i(t)$ the depreciation rate.

Utility and values

Discount rate: β
Current period utility: no consumption smoothing, logarithmic utility

$$u_i(t) = \delta_i(t) + log\, H_i(t) - \left(\tau_i(t) + c_i \frac{\tau_i(t)^2}{2}\right)$$

Ben Porath's formulation: $c_i = 0$.

- c_i : additional cost of effort to invest in human capital.

Value of human capital at the terminal period:

$$W_R(H_i(T)) = \delta^* + \kappa_i \log H_i(T), \text{ (Reduced form)}$$

Remark: the capitalized return on investments**,** κ_i is assumed to be lower than the "standard" capitalized return of a constant flow of one euro:

$$\kappa_i = \frac{1}{1 - \beta \Pr(\text{Survival})} \leq \frac{1}{1-\beta}$$

The model is a function of four individual structural parameters, ρ_i, c_i and κ_i as well as the initial level of human capital stock.

The evolution of human capital investments

Proposition: *When:*

$$\beta \rho_i \kappa_i > 1,$$

human capital investments are:

$$\tau_i(t) = \frac{1}{c_i} \left\{ \rho_i \left[\frac{\beta}{1-\beta} + \beta^{T+1-t} (\kappa_i - \frac{1}{1-\beta}) \right] - 1 \right\} > 0, \quad \forall t < T+1$$

- Investments decay (deterministically) over time at an exponential rate.

Remark1: Future investments can be predicted: no available information to distinguish unobserved heterogeneity from uncertainty (as in Cunha, Heckman and Navarro, 2007).

Remark2: Condition $\beta \rho_i \kappa_i > 1$ ensures that investments are positive until period $T+1$.
If this is not the case, there is optimal stopping. Human capital invetsments stop anytime before period $T+1$ according to the value of ρ_i and κ_i.

Optimal stopping

Proposition: *The investment sequence is such that for any* $t \in [1, T]$

$$\forall t \geq S_i, \tau_i(t) = 0, \text{ and } \tau_i(T_i - 1) > 0$$

if and only if:

$$\frac{1}{\kappa_{i,T_i-1}} < \beta\rho_i \leq \frac{1}{\kappa_{i,T_i}}, \tag{1}$$

where:

$$\kappa_{it} = \frac{1}{1-\beta} + \beta^{T-t}(\kappa_i - \frac{1}{1-\beta})$$

- *Additionally, when condition (1) is satisfied we can replace period* $T+1$ *by period* $T_i = t+1$ *in the equation deriving human capital investments before and including period* $T_i - 1$.
- *Period* T_i *is the optimal stopping period for human capital investments and:*

$$\tau_i(t) = \frac{1}{c_i}\left\{\rho_i\left[\frac{\beta}{1-\beta} + \beta^{T_i-t}(\kappa_{i,T_i} - \frac{1}{1-\beta})\right] - 1\right\} > 0, \;\; \forall t < T_i.$$

The Lifecycle Profile of Earnings

Human capital dynamics:

$$H_i(t) = H_i(1) \exp\left[\sum_{l=1}^{t-1} \rho_i \tau_i(l) - \sum_{l=1}^{t-1} \lambda_i(l)\right] \text{ for } t \geq 2.$$

implies that the logarithm of earnings in period t can be written as a three factor model:

$$\log y_{it} = \eta_{i1} + \eta_{i2} t + \eta_{i3} \beta^{-t} + v_{it},$$

where v_{it} is the (log) price of human capital net of cumulative depreciation .

Remark 1: Above $\tau_i(t) \geq 0$ for all t in the period of observation. If this is not the case, $\tau_i(t) = 0$ and:

$$log\, y_i(t+1) = log\, y_i(t) + v_{it+1} - v_{it}.$$

The former or latter specification apply according to values of structural parameters.

Remark 2: The model accomodates the two most popular specifications for dynamic models of earnings: random growth and restricted income profile specifications (see e.g. Guvenen, 2009).

Homogenous and heterogenous parameters

Heterogeneous parameters

- $\log H_i(0)$, initial level of human capital e.g. schooling (ability to earn)
- ρ_i : returns on investments (learning ability)
- c_i: cost of effort (learning ability)
- κ_i: terminal value of human capital (ability to earn)

Homogeneous parameters:

- β: discount rate

Restricted stochastic processes :

- $v_{it} = (\delta_{it} - \sum_{l=1}^{t-1} \lambda_{il})$: (log) price of human capital net of cumulative depreciation

Assumption: From now on, we assume that everybody invests until the last period of observation.

Reduced form identification

- *Between groups* g determined by explanatory variables (age of entry, skills). Restrictions on "market prices", v_{gt} in:

$$\overline{\log y}_{gt} = \overline{\eta}_{g1} + \overline{\eta}_{g2}t + \overline{\eta}_{g3}\beta^{-t} + v_{gt},$$

$$v_{gt} \perp (1, t, \beta^{-t}), v_{gt} \perp (\overline{\eta}_{g1}, \overline{\eta}_{g2}, \overline{\eta}_{g3}).$$

  in which the sign $\perp$ denotes mean independence. Implicitly, expectations are perfect.
  Depends also on whether productivity or earnings profile is attributable to human capital only or to other factors (physical capital for instance). This view influences how we deflate mean earnings by inflation or inflation+labor productivity increases over the period.

- *Within groups*: Restrictions that frictions, $v_{it} - v_{gt}$, are uncorrelated with factors & factor loadings:

$$v_{it} - v_{gt} \perp (1, t, \beta^{-t}), v_{it} - v_{gt} \perp (\eta_{i1} - \overline{\eta}_{g1}, \eta_{i2} - \overline{\eta}_{g2}, \eta_{i3} - \overline{\eta}_{g3}).$$

Rem: Between and within group specification have not the same dynamics. We treat them separately and recompose these dimensions to get estimates of $\eta_{i1}, \eta_{i2}, \eta_{i3}$.

Structural to reduced form transformation

Reduced form factor loadings:

$$
\begin{aligned}
\eta_{i1} &= \log H_i(1) - \frac{\rho_i^2}{c_i}\left(\kappa_i - \frac{1}{1-\beta}\right)\frac{\beta^{T+2}}{1-\beta} - \frac{\rho_i+1}{c_i}\left(\rho_i\frac{\beta}{1-\beta} - 1\right), \\
\eta_{i2} &= \frac{\rho_i^2}{c_i}\frac{\beta}{1-\beta} - \frac{\rho_i}{c_i}, \\
\eta_{i3} &= \frac{\rho_i^2}{c_i}\left(\kappa_i - \frac{1}{1-\beta}\right)\frac{\beta^{T+2}}{1-\beta} - \frac{\rho_i}{c_i}\beta^{T+1}(\kappa_i - \frac{1}{1-\beta}).
\end{aligned}
$$

Structural restrictions

Two types of restrictions:

- On the capitalized rate after the terminal period:

$$0 \leq \kappa_i \leq \frac{1}{1-\beta}$$

- On human capital investments $\tau_i(t) \geq 0$.

This yields 3 restrictions:

$$\begin{aligned} \eta_{3i} &< 0, (\text{i.e. } \kappa_i \leq \frac{1}{1-\beta}) \\ \pi_T(\beta)\eta_{2i} + \eta_{3i} &> 0, (\text{i.e. } 0 \leq \kappa_i) \\ \eta_{2i} &> 0, (\text{i.e. } \tau_i(t) \geq 0) \end{aligned}$$

where $\pi_T(\beta)$ is a function of β and the last date of investment.

Reduced form to structural parameters

Inversion of the previous system: Three equations for four structural parameters.
κ_i is point identified although ρ_i and c_i are only partially identified (only lower bounds are identified).

Capitalization factor after retirement:

$$\kappa_i = \frac{1}{1-\beta} + \beta^{-(T+1)}\frac{\eta_{i3}}{\eta_{i2}},$$

Costs and returns:

$$\rho_i > \rho_L, c_i > c_L,$$

$$c_i = c(\rho_i).$$

Data

- Administrative data extracted from Social Security records. Restricted to the private sector. Few observable characteristics: age of entry and a skill variable for the first job.
- A single entry cohort entering the labor market in 1977 observed until 2007. Missing years in the data: 81, 83 and 90. The entry is defined at the date of the first permanent job.
- *Additional sample selection:* We selected males present in 77, 78, 82 and 84 having full time jobs. Number of observations = 7447
- Missing data are assumed to be missing at random. Entries and reentries. Roughly half of the sample is present at the end of the sampling period.

Life Cycle Profile of Mean Earnings

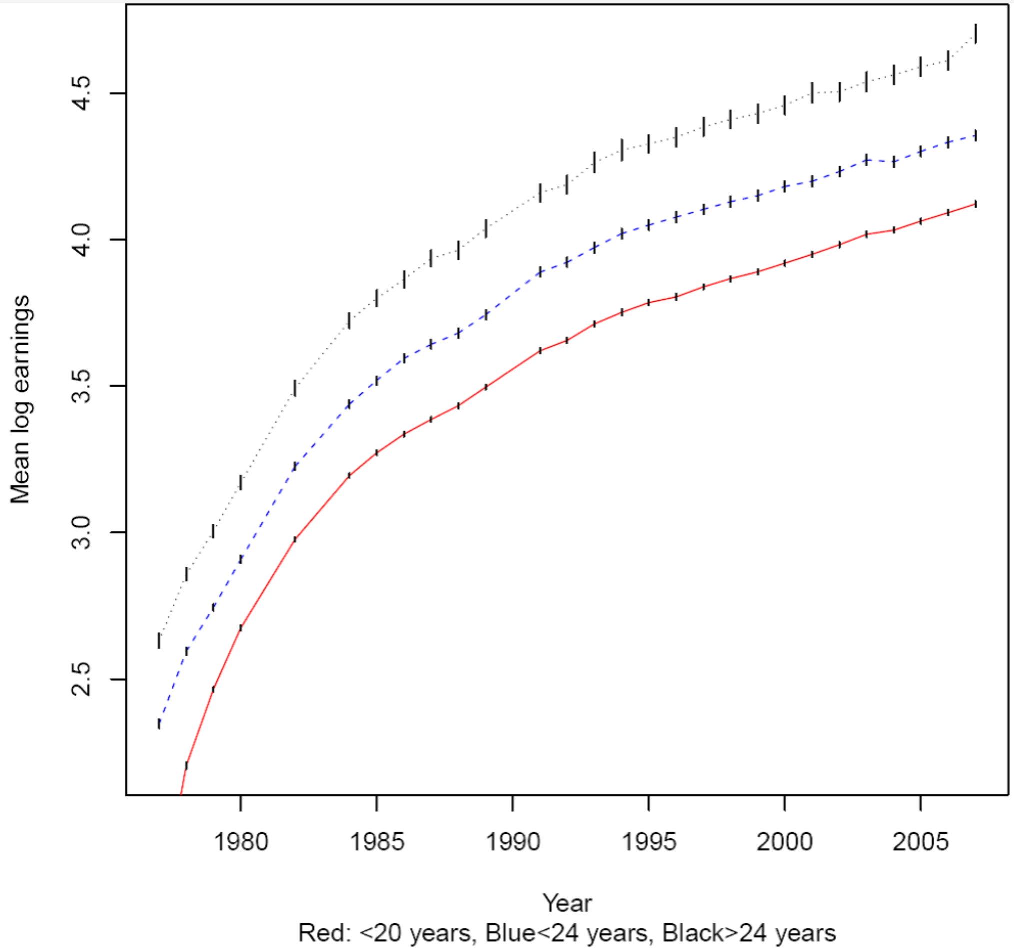
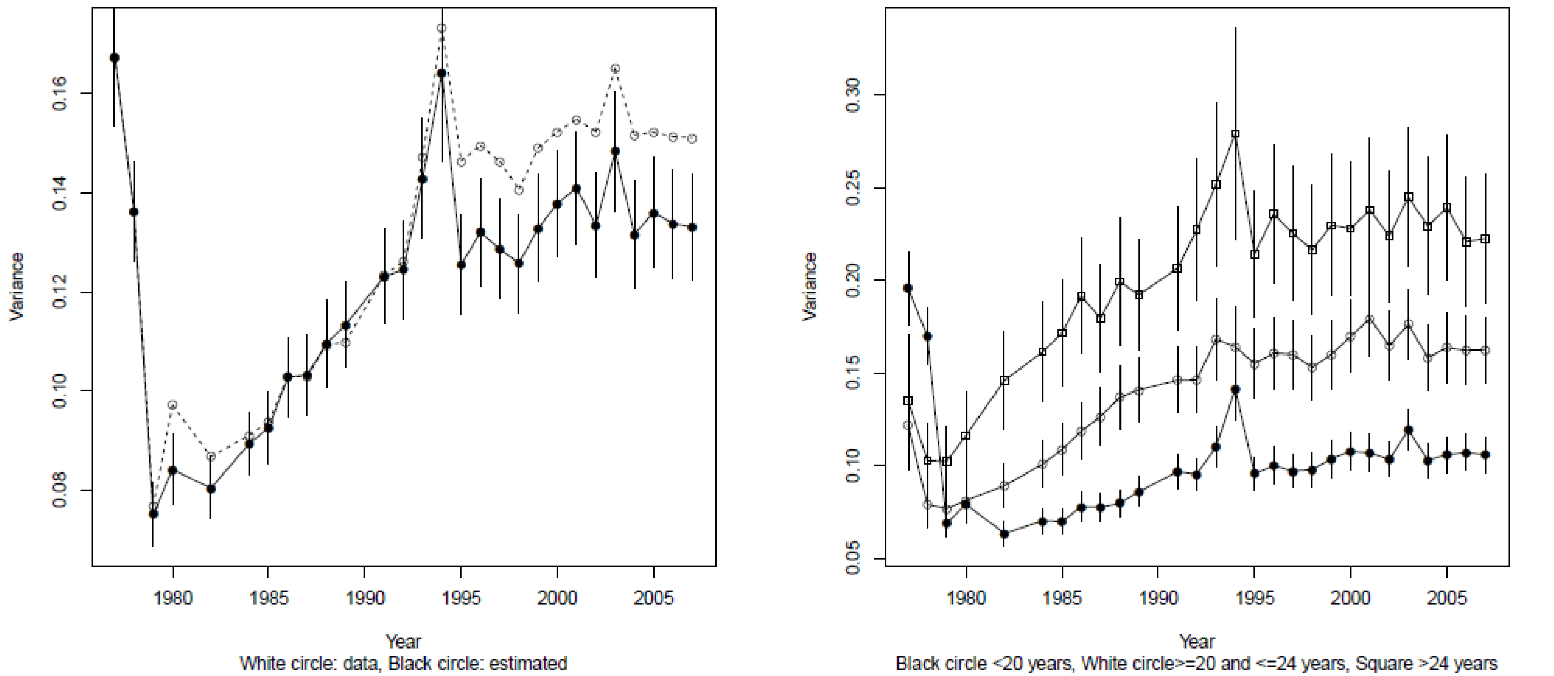


Figure 1: Mean log earnings by age at entry: 1977-2007

Life Cycle Variances



Note: The small lines represent the 95% confidence intervals.

(A) full sample (B) by age of entry

Figure 2: Cross-sectional variance of earnings: 1977-2007

Quite similar to the same cohorts in the US: Rubinstein and Weiss (2006).

Autocorrelations

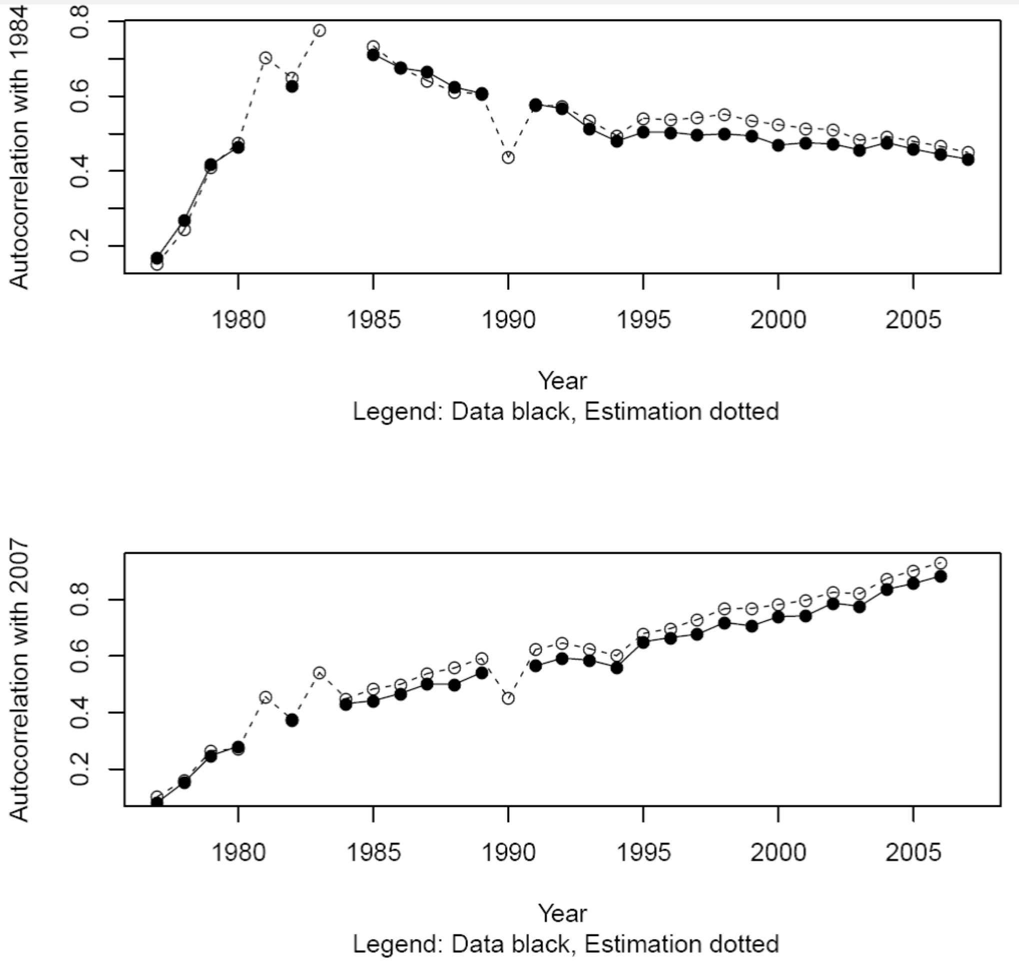


Figure 3: Autocorrelations with 1986 and 2007

Empirical strategy

(A1) *Human capital investments are positive until the end of the period of observation*

(A2) *The discount factor* β *is set to* $.95$

(A3) *Within group estimation in two stages:*

1. **Random effect estimation** identifying the variances and covariances of individual effects and idiosyncratic individual-period terms (including initial conditions). This allows for limited counterfactual analysis since this is limited to policies affecting individual effects **linearly only**.
2. **Fixed effect estimation** using the previous random effect estimates to filter out autocorrelation. Nevertheless, the individual effect estimates are biased in $1/T$. Our strategy is to assess the bias using the length of the observation period and the random effect benchmark. The bias becomes "reasonable" when the number of observations in individual profiles, $N_i > 20$.

(A4) *Between group estimation by OLS in each group*
This procedure combined with fixed effect estimation allows for testing and non linear counterfactual analysis

Random Effect Estimation

- *Covariance structures*: Abowd and Card, 1989 , by minimum distance
- *GMM*: for instance Alvarez and Arellano, 2003, and optimal choice of moments, Okui, 2009 (also empirical likelihood).
- *Bias corrected estimates in dynamic panel data*: Hahn and Kuersteiner, 2002,
- *Bayesian analyses*: Geweke and Keane, 2000, Hirano, 2002.

This paper:

- *Identification of the covariance matrix of individual effects:* Factors are supposed to be known $(1, t, 1/\beta^t)$ and the idiosyncratic shocks have a finite ARMA-type structure. Then the factor structure is identiied as well as the covariance of the idiosyncratic shocks (Arellano and Bonhomme, 2010)
- *Pseudo likelihood estimates:* Alvarez and Arellano, 2004. Very much adapted to the missing data structure in our dataset.

Random Effect Specification

Earnings equation:

$$u_{it} = \eta_{i1} + \eta_{i2}t + \eta_{i3}\frac{1}{\beta^t} + v^c_{it} \text{ for any } t = 1, ., T.$$

where u_{it} is the residual of the (fully saturated) regression of $\log y_{it}$ on covariates and periods and η^c_i is the centered (e.g. deviation from the mean) version of the ηs:

$$v^c_{it} = \alpha_1 v_{i(t-1)} + ... + \alpha_p v_{i(t-p)} + \sigma_t w_{it},$$

and:

$$w_{it} = \zeta_{it} - \psi_1 \zeta_{it-1} - ... - \psi_q \zeta_{it-q}.$$

$(\eta^c_{i1}, \eta^c_{i2}, \eta^c_{i3})$ are independent of idiosyncratic shocks $\zeta_{iT}, ., \zeta_{i(1-q)}$.

The model is *incomplete* because of initial conditions ($p \geq 1$).
Initial conditions of the process $(u_{i(1-p)}, ., u_{i0})$ are freely correlated with $(\eta^c_{i1}, \eta^c_{i2}, \eta^c_{i3})$ and with predetermined shocks in $\zeta_{i0}, ., \zeta_{i(1-q)}$ (i.e. the entry process is not generated by the same stochastics).

Arguments of the (pseudo) likelihood function:

$$\alpha, \psi, \sigma, V\eta^c_i, V\zeta_i, Vy_{i0}, Cov(y_{i0}, \zeta_i), Cov(y_{i0}, \eta^c_i).$$

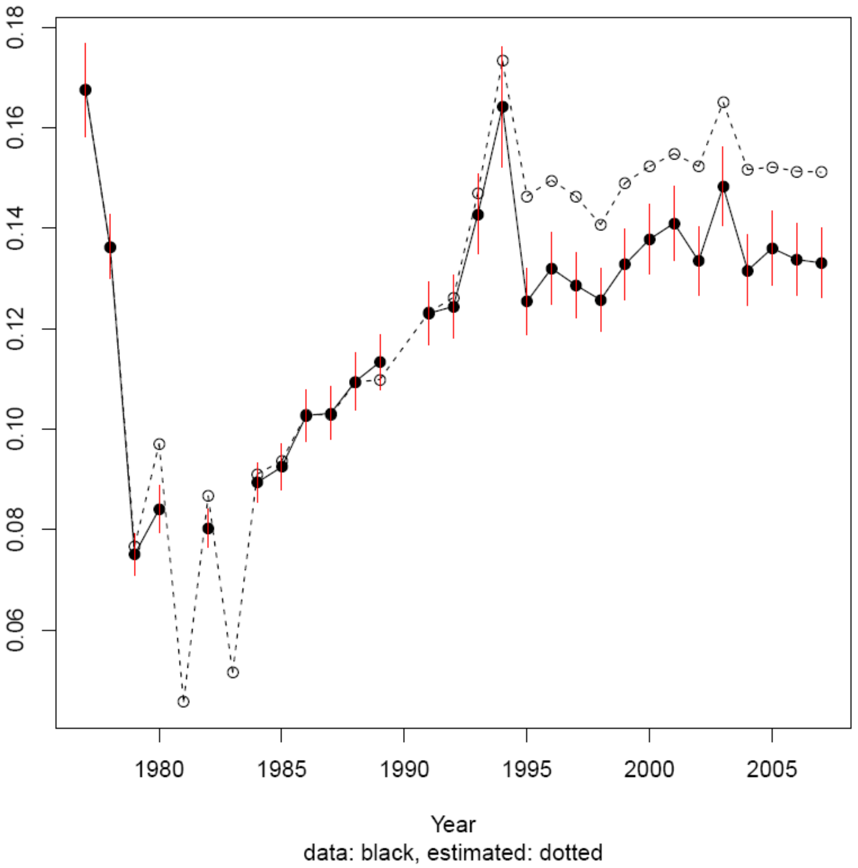
Random effect estimation: factors

	1-1	1-2	1-3	2-1	2-2	2-3	3-1	3-2	3-3
σ_{η_1}	.302	.302	.301	.310	.306	.304	.306	.300	.298
	(.001)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.003)	(.004)
σ_{η_2}	.038	.039	.039	.038	.039	.036	.038	.037	.037
	(.005)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
σ_{η_3}	.255	.259	.256	.263	.260	.248	.258	.247	.242
	(.005)	(.006)	(.006)	(.004)	(.005)	(.005)	(.005)	(.006)	(.007)
ρ_{η_1,η_2}	.473	.413	.454	.571	.486	.610	.505	.485	.365
	(.016)	(.021)	.021	(.013)	(.017)	(.013)	(.017)	(.020)	(.030)
ρ_{η_1,η_3}	- .604	- .548	- .586	- .694	- .618	- .729	- .636	- .620	- .509
	(.003)	(.020)	.019	(.011)	(.015)	(.012)	(.016)	(.019)	(.029)
ρ_{η_2,η_3}	- .946	- .948	- .947	- .945	- .946	- .941	- .946	- .943	- .944
	(.023)	(.003)	.003	(.002)	(.002)	(.003)	(.002)	(.003)	(.004)

Remark: Sign of the correlations:

- η_1, η_2 : Level and growth positive correlation in the long run (not in the short-run because of initial conditions)
- η_3 with η_1, η_2 : The larger the level or slope the larger the final decrease.

Goodness of fit: Variances



(A) full sample

Figure 2: Cross-sectional variance

Between Estimation: Quantity/Prices

We use:

$$\overline{\log y}_{gt} = \overline{\eta}_{g1} + \overline{\eta}_{g2}t + \overline{\eta}_{g3}\beta^{-t} + v_{gt},$$

$$v_{gt} \perp (1, t, \beta^{-t}),$$

estimate by OLS, group by group, and use HAC standard errors.

Procedures below are (reasonably) robust to various departures from this procedure (including a productivity deflated mean wage $\overline{\log y}_{gt}$).

Fixed effects: Within Estimation

Writing:

$$u_i^{[1-p,T]} = D\eta_i^c + w_i^{[1-p,T]} \text{ where } D = M(\beta)^{[1-p,T]} + C,$$

where C eliminates the correlation between η_i^c and $w_i^{[1-p,T]}$

$$C = E(v_i^{[1-p,T]}\eta_i^{c\prime})(V(\eta_i^c))^{-1}.$$

Replace D by the random effect estimate $\hat{D}$, use the random effect estimate for the covariance matrix of w, Ω_w and compute the FGLS estimate of individual η_i^c as:

$$\hat{\eta}_i^c = \hat{B}y_i^{[1-p,T]},$$

in which $\hat{B}$ is the estimate of:

$$B \;= (D'\Omega_w^{-1}D)^{-1}D'\Omega_w^{-1}.$$

Rem: Below, we add mean estimates $\overline{\eta}_{g1}$ to within estimates to derive full estimates of η_i.

Fixed effect estimates: Remarks

Remark 1: An unfeasible estimate is:

$$\tilde{\eta}_i^c = B u_i^{[1-p,T]} = \eta_i^c + B w_i.$$

and we have (e.g. Arellano and Bonhomme, 2010):

$$\begin{aligned} V(\tilde{\eta}_i^c) &= EV(\tilde{\eta}_i^c \mid \eta_i^c) + VE(\tilde{\eta}_i^c \mid \eta_i^c) \\ &\Longrightarrow V(\tilde{\eta}_i^c) = B\Omega_w B^T + V\eta_i^c, \end{aligned}$$

where the $1/T$ bias is $B\Omega_w B^T$.

Remark 2: Our estimate is:

$$\hat{\eta}_i^c = \hat{B} u_i^{[1-p,T]} = \tilde{\eta}_i^c + (\hat{B} - B) u_i^{[1-p,T]}.$$

We compute HAC standard errors, grouped by the number of observations, N_i. (Robust if we use individual specific variances)

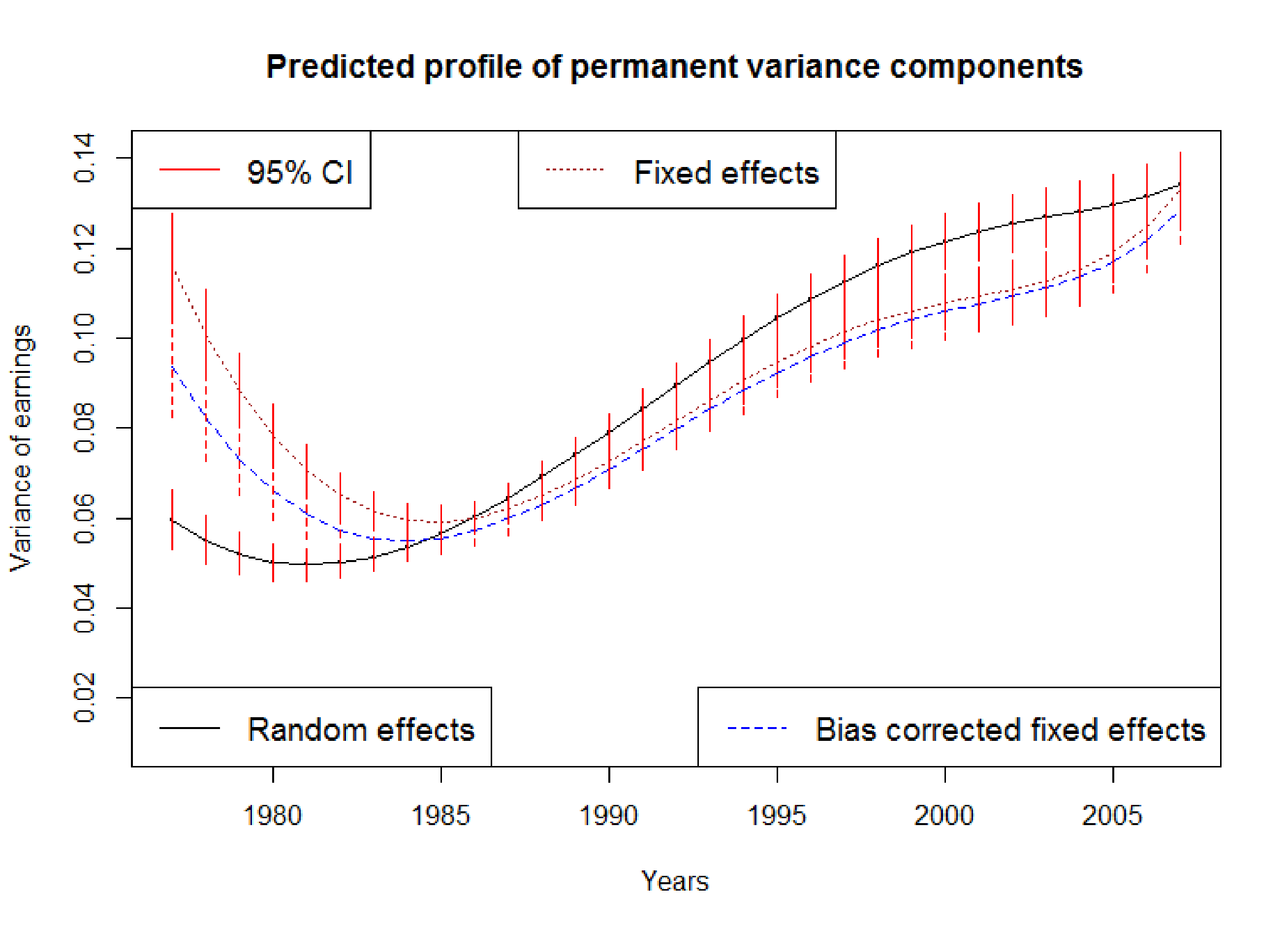
Random and fixed effect estimates of individual effect covariances

Sample periods	$Var(\eta_1)$	$Cov(\eta_1, \eta_2)$	$Cov(\eta_1, \eta_3)$	$Var(\eta_2)$	$Cov(\eta_2, \eta_3)$	$Var(\eta_3)$
(3,15]	11	0.93	-12	0.093	-1.1	14
	(16)	(1.3)	(18)	(0.1)	(1.4)	(19)
(15,22]	0.5	0.057	-0.57	0.01	-0.09	0.83
	(0.08)	(0.01)	(0.11)	(0.0016)	(0.015)	(0.15)
(22,26]	0.14	0.011	-0.099	0.0038	-0.027	0.2
	(0.0073)	(0.0011)	(0.0091)	(0.00032)	(0.0024)	(0.018)
(26,28]	0.076	0.0043	-0.038	0.002	-0.013	0.09
	(0.0039)	(0.00058)	(0.0041)	(0.00015)	(0.00098)	(0.0067)
Complete sample	2.6	0.22	-2.8	0.024	-0.27	3.3
	(3.5)	(0.28)	(3.8)	(0.023)	(0.31)	(4.2)
Random effects	0.093	0.0059	-0.05	0.0015	-0.0093	0.066
	(0.0036)	(0.00051)	(0.004)	(0.00011)	(0.00079)	(0.0059)

Notes: The first four lines are obtained using fixed effect estimates. Sample periods = number of observed periods. Standard errors

(sampling and parameter uncertainty, 1000 MC simulations) between brackets.

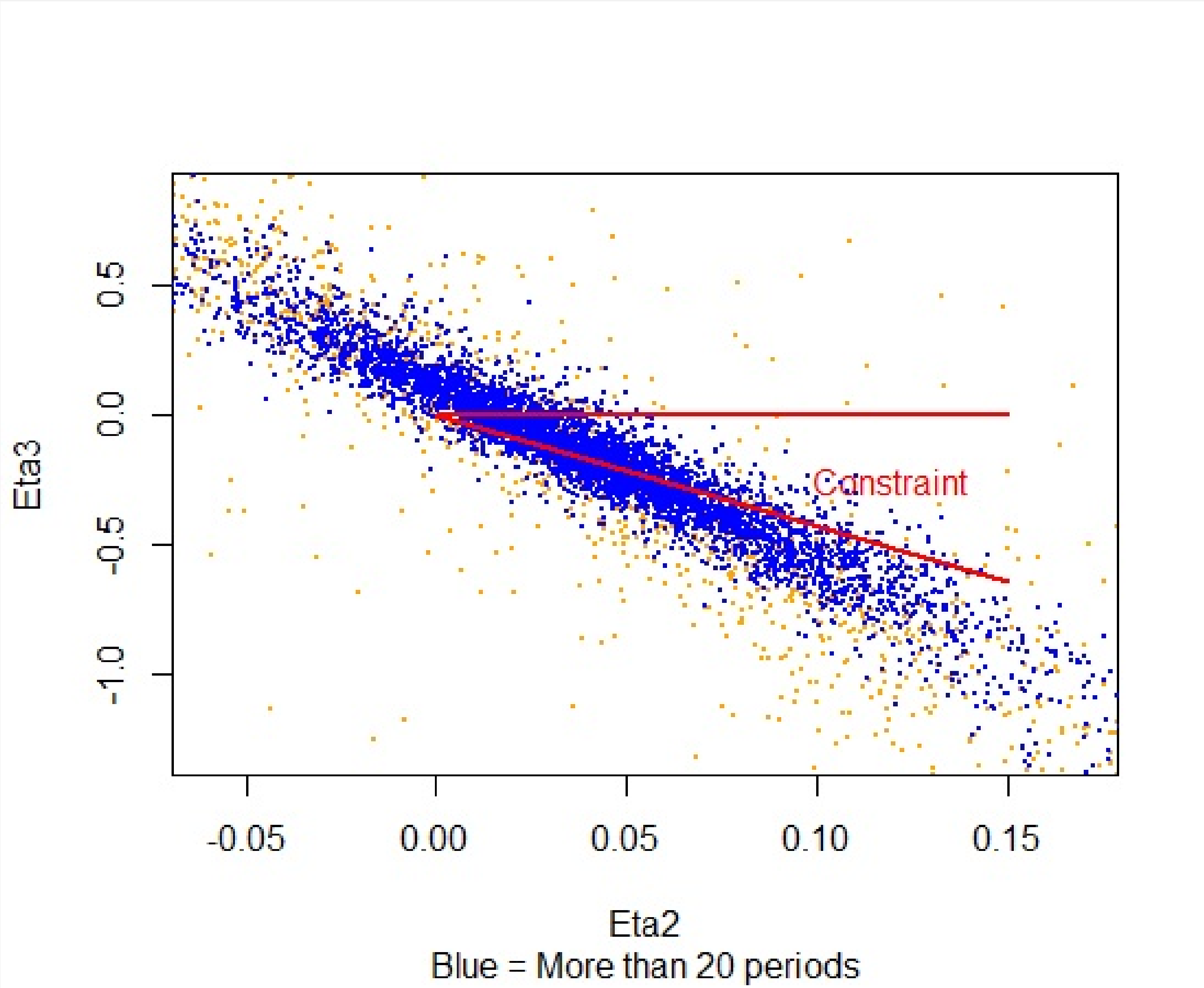
Random and fixed effect estimates of variance profiles



Abstracting from the idiosyncratic noise of transitory earnings

Earning variance profile due to heterogeneity terms only: $V(Matrix(1, t, \beta^{-t})\eta_i)$.

Structural restrictions on FE estimates



Violations of restrictions by FE estimates

(single dimensional: frequency of rejections at 5%)

Sample periods	$\eta_2 > 0$	$\eta_3 < 0$	$\eta_3 + \lambda\eta_2 < 0$
(3,15]	0.064	0.068	0.086
	(0.033)	(0.039)	(0.037)
(15,22]	0.09	0.11	0.13
	(0.015)	(0.017)	(0.016)
(22,26]	0.053	0.094	0.15
	(0.0094)	(0.014)	(0.017)
(26,28]	0.032	0.074	0.14
	(0.0081)	(0.015)	(0.02)

Notes: Sample periods = number of observed periods. 5 per cent level rejection frequency of single-dimensional restrictions. Standard errors (sampling and parameter uncertainty, 1000 MC simulations) between brackets.

Constrained estimates

GLS under constraints also corresponds to the maximization of the pseudo-likelihood function :

$$L(\eta_i \mid \hat{\eta}_i) = H(\hat{\eta}_i).\exp\left(-\frac{1}{2}(\eta_i - \hat{\eta}_i)'\Omega_\eta^{-1}(\eta_i - \hat{\eta}_i)\right) L_0(\eta_i),$$

in which Ω_η is the variance of unconstrained ηs and constraints are imposed through the prior distribution, $L_0(\eta_i)$.

Equivalent to solve:

$$\min_{\eta_i}(\eta_i - \hat{\eta}_i)'\Omega_\eta^{-1}(\eta_i - \hat{\eta}_i)$$

under the constraints:

$$\eta_{i2} > 0, \eta_{i3} < 0, \eta_{i3} > -\lambda\eta_{i2}.$$

Remark: All components are affected (even the unrestricted η_1). Ω_η^{-1} is estimated using the estimated variance of the ηs (recomposing between and within group dimensions).

Distance (QLR statistic):

$$d_i = (\tilde{\eta}_i - \hat{\eta}_i)'\Omega_\eta^{-1}(\tilde{\eta}_i - \hat{\eta}_i)$$

Violations of restrictions (global)

Sample periods	P-values $<$0.10	0.05	0.01
(3,15]	0.16	0.12	0.078
	(0.012)	(0.011)	(0.0084)
(15,22]	0.21	0.17	0.12
	(0.011)	(0.0096)	(0.0083)
(22,26]	0.21	0.17	0.12
	(0.0088)	(0.0081)	(0.007)
(26,28]	0.18	0.15	0.1
	(0.0085)	(0.0078)	(0.0066)
Complete sample	0.19	0.15	0.1
	(0.005)	(0.0045)	(0.0038)

Notes: Sample periods = number of observed periods. Frequency of p-values associated to the test of restrictions satisfying the conditions. Standard errors (sampling and parameter uncertainty, 20 Monte Carlo simulations) between brackets. Statistic distribution obtained by 150 replications.

Simulations

Constrained estimates can be at the frontier of the constrained set and this leads to implausible structural estimates. (in economic terms).

Idea:

To smooth constrained estimates, draw simulations into the constrained distribution (with some additional trimming to avoid frontier points):

$$L(\eta_i \mid \hat{\eta}_i) = H(\hat{\eta}_i). \exp\left(-\frac{1}{2}(\eta_i - \hat{\eta}_i)' \Omega_\eta^{-1} (\eta_i - \hat{\eta}_i)\right) L_0(\eta_i)$$

where the prior distribution $L_0(\eta_i)$ translates structural constraints.

Drawing in a bivariate truncated distribution: Gibbs sampling.

Counterfactuals: Construction

"Technological" improvement in survival probabilities: Additional K years after period T during which the survival probability remains equal to 1 equivalent to prolonging life expectancy by K years.
This amounts to transforming κ_i into κ_i^*:

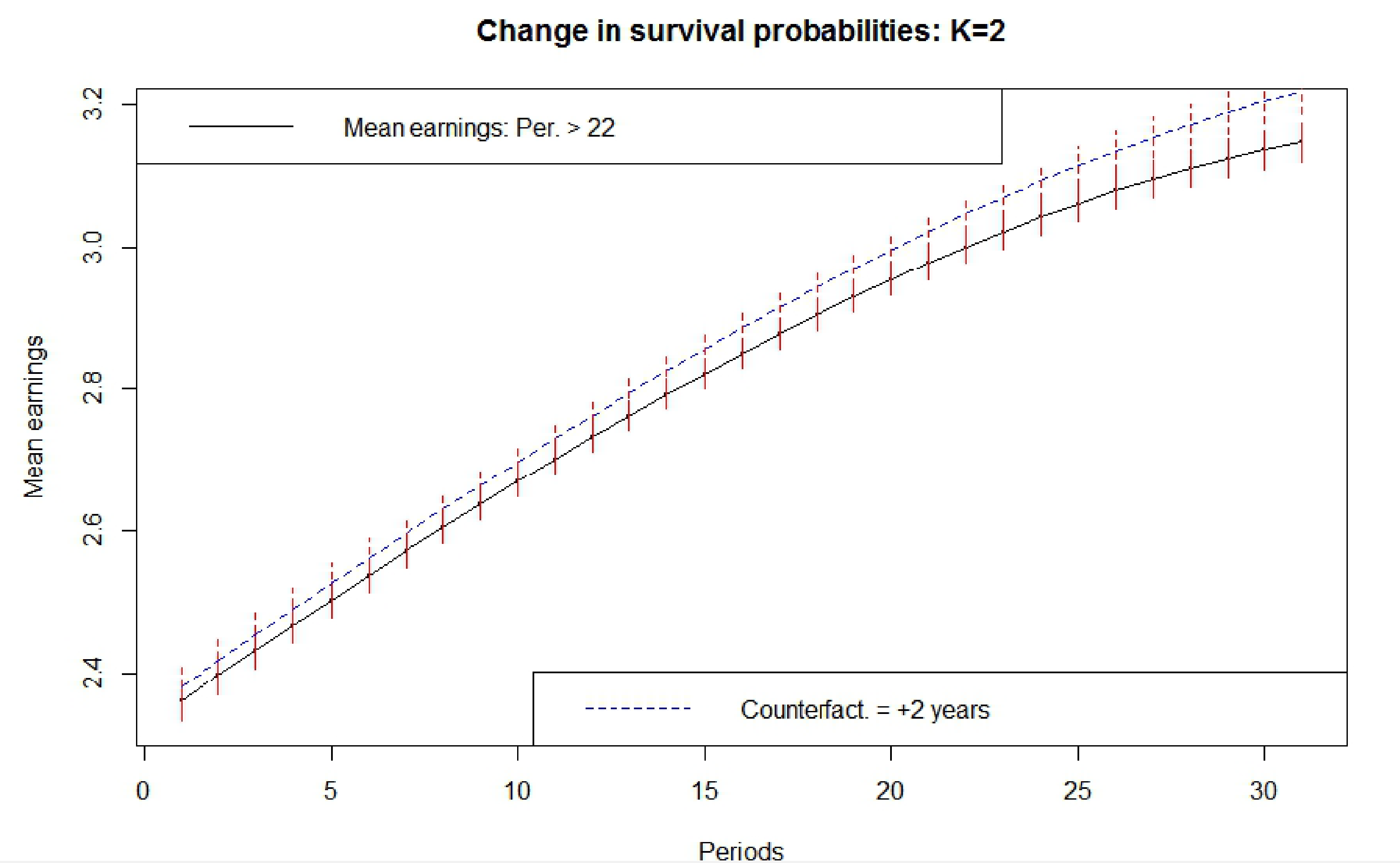
$$\kappa_i^* - \frac{1}{1-\beta} = \beta^K(\kappa_i - \frac{1}{1-\beta})$$

Other parameters ρ_i and c_i are held fixed.
News revealed at time $t = 1$ so that the initial level of human capital would remain the same. We assume that there is infinite demand for human capital at the rental prices that were effectively observed as well as decumulation shocks so that the transitory earning process also remains the same .
The new values $(\eta_{1i}^*, \eta_{2i}^*, \eta_{3i}^*)$ are such that $\eta_{2i}^* = \eta_{2i}$, $\eta_{i3}^* = \beta^K \eta_{i3}$ and:

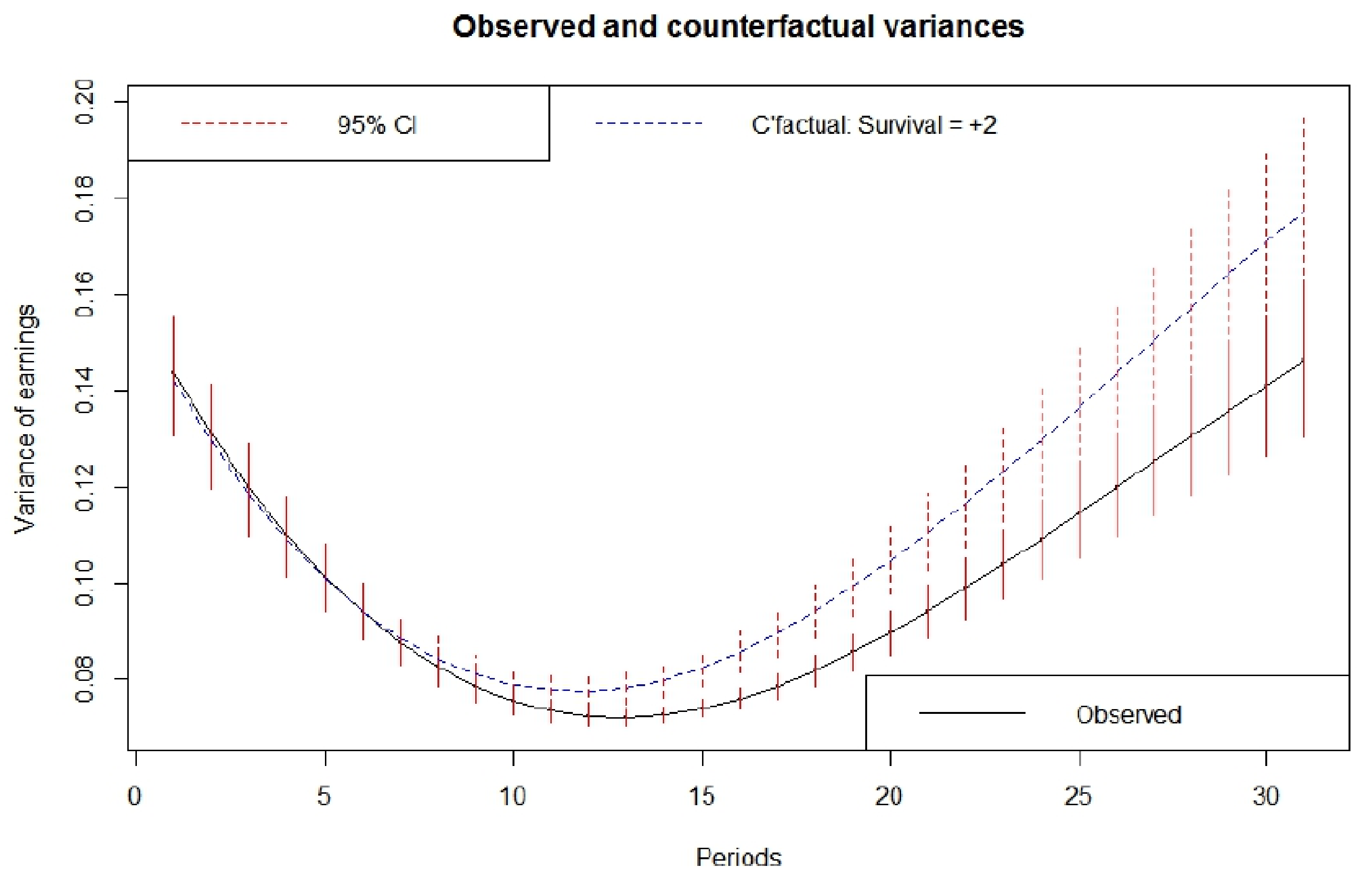
$$\eta_{i1}^* - \eta_{i1} = -\frac{\rho_i^2}{c_i}\left(\kappa_i - \frac{1}{1-\beta}\right)\frac{\beta^{T+2}}{1-\beta}(\beta^K - 1).$$

Parameters ρ_i and c_i are not identified and only a lower bound (ρ_i^L, c_i^L) on their values can be computed. For this simulation, assumption that $\rho_i = \rho_i^L$, $\rho_i^L = 1.20\rho_i$ etc to assess the robustness of this construction.

Counterfactuals: change in survival probabilities (1)



Counterfactuals: change in survival probabilities (2)



Note: Sample of 4292 observations for which observed periods>22. Standard errors are due to sampling and parameter uncertainty (30 Monte Carlo replications)

Conclusion and extensions

- An empirically tractable theoretical model of earnings profiles
- A combination of random and fixed effect methods
- Testing individual structural restrictions and computing counterfactuals

- Extension to different education groups
- Departures from the missing at random assumption
- Joint financial and human capital accumulation
- Mixture model between factor model (HIP) and restricted income profiles (RIP)

Other slides

Attrition

	1977	1978	1979	1980	1985	1986	1987	1988	1989	1991	1992	1993	1994	1995
1977	1													
1978	1	1												
1979	.882	.882	.882											
1980	.868	.868	.786	.868										
1982	1	1	.882	.868										
1984	1	1	.882	.868										
1985	.849	.849	.751	.743	.849									
1986	.834	.834	.739	.731	.75	.834								
1987	.804	.804	.714	.704	.718	.737	.804							
1988	.765	.765	.675	.668	.694	.690	.691	.765						
1989	.777	.777	.689	.677	.701	.694	.691	.689	.777					
1991	.743	.743	.658	.65	.67	.663	.655	.649	.678	.743				
1992	.736	.736	.653	.647	.663	.655	.649	.642	.662	.679	.736			
1993	.749	.749	.665	.653	.657	.666	.654	.631	.652	.659	.673	.749		
1994	.581	.581	.515	.506	.508	.518	.511	.492	.506	.513	.517	.544	.581	
1995	.725	.725	.643	.634	.636	.644	.632	.609	.628	.63	.635	.661	.535	.725
1996	.721	.721	.641	.631	.631	.638	.627	.603	.622	.622	.627	.652	.521	.671
1997	.71	.71	.629	.621	.622	.63	.619	.596	.613	.612	.618	.642	.511	.649
1998	.708	.708	.628	.619	.618	.625	.615	.591	.61	.609	.614	.636	.506	.642
1999	.708	.708	.628	.617	.617	.623	.614	.59	.61	.605	.609	.63	.502	.635
2000	.701	.701	.622	.611	.612	.62	.61	.583	.6	.595	.601	.623	.497	.625
2001	.687	.687	.61	.598	.599	.605	.595	.573	.589	.584	.587	.605	.479	.608
2002	.67	.67	.595	.586	.588	.591	.581	.559	.575	.568	.573	.592	.471	.59
2003	.616	.616	.547	.539	.544	.542	.532	.516	.533	.526	.53	.539	.425	.538
2004	.63	.63	.559	.551	.552	.556	.545	.523	.541	.534	.539	.555	.441	.555
2005	.634	.634	.560	.552	.554	.558	.548	.526	.544	.536	.541	.558	.446	.557
2006	.634	.634	.561	.553	.556	.557	.549	.525	.544	.535	.541	.556	.444	.553
2007	.627	.627	.557	.547	.55	.552	.542	.521	.538	.531	.535	.548	.436	.547

Frequencies of observations present in the sample relative to the full sample

Random effect estimation: Akaike criterion

Table 6: AIC criterion

ARMA(p,q)	q=1	q=2	q=3
p=1	-344885	-344899	-344906
	(43)	(45)	(47)
p=2	-345301	-345447	-345733
	(47)	(50)	(53)
p=3	-345839	-346133	-346293
	(51)	(54)	(58)

AIC criterion computed as -2log(L)$+2K$, with L the likelihood and K the number of parameters. Number of parameters in brackets.

Random effect estimation: dynamics

	1-1	1-2	1-3	2-1	2-2	2-3	3-1	3-2	3-3
α_1	.702	.729	.711	.263	.186	.220	.200	.203	.194
	(.005)	(.006)	(.007)	(.011)	(.011)	(.011)	(.012)	(.011)	(.011)
α_2				.145	.324	.143	.191	.143	.161
				(.004)	(.008)	(.009)	(.005)	(.009)	(.009)
α_3							.022	.087	.187
							(.003)	(.004)	(.008)
ψ_1	.369	.391	.373	- .091	- .172	- .135	- .164	- .166	- .189
	(.005)	(.005)	(.007)	(.011)	(.011)	(.012)	(.012)	(.011)	(.011)
ψ_2		.020	.017		.170	- .028		- .046	- .046
		(.003)	(.003)		(.006)	(.008)		(.008)	(.008)
ψ_3			- .012			- .080			.114
			(.004)			(.004)			(.007)

Far from a unit root!

Model selection: Akaike indicates ARMA(3,3), our preferred specification is ARMA(3,1)

Random effect estimation: Initial conditions

	1-1	1-2	1-3	2-1	2-2	2-3	3-1	3-2	3-3
$cov(\eta_1, y_0)$	- .227	- .257	- .237	- .156	- .214	- .149	-.186	- .201	- .282
	(.019)	(.017)	.017	(.015)	(.016)	(.016)	(.016)	(.017)	(.019)
$cov(\eta_1, y_{-1})$				- .127	- .183	- .113	- .153	- .168	- .253
				(.016)	(.017)	(.017)	(.017)	(.018)	(.020)
$cov(\eta_1, y_{-2})$							- .169	- .185	- .267
							(.018)	(.019)	(.022)
$cov(\eta_2, y_0)$	.358	.402	.374	.232	.335	.155	.219	.253	.361
	(.022)	(.020)	.021	(.017)	(.019)	(.021)	(.020)	(.022)	(.026)
$cov(\eta_2, y_{-1})$				.218	.331	.119	.242	.235	.352
				(.019)	(.021)	(.024)	(.022)	(.025)	(.029)
$cov(\eta_2, y_{-2})$							.239	.253	.351
							(.024)	(.027)	(.032)
$cov(\eta_3, y_0)$	- .290	- .333	- .305	- .179	- .270	- .107	- .163	- .195	- .291
	(.018)	(.023)	.023	(.020)	(.022)	(.023)	(.023)	(.024)	(.029)
$cov(\eta_3, y_{-1})$				- .169	- .272	- .077	- .190	- .181	- .287
				(.021)	(.023)	(.025)	(.023)	(.027)	(.032)
$cov(\eta_3, y_{-2})$							- .181	- .194	- .282
							(.026)	(.029)	(.035)

Remark: Investment model suggests a negative correlation between wage level and wage growth at the beginning of the life cycle and a positive correlation late in workers'career (Rubinstein and Weiss, 2006). This is not the case and growth is positively correlated with initaila level conditions (but less that "permanent" levels). Even more surprising is the negative correlation of levels and initial conditions.

Random effect estimation: Period heteroskedasticity

	1-1	1-2	1-3	2-1	2-2	2-3	3-1	3-2	3-3
1978	.311	.312	.312						
	(.001)	(.002)	(.002)						
1979	.254	.257	.255	.222	.232	.219			
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)			
1980	.223	.223	.223	.222	.227	.221	.224	.224	.230
	(.005)	(.001)	(.001)	(.001)	(.001)	(.001)	(.002)	(.002)	(.002)
1981	.264	.260	.263	.000	.103	.002	.004	.006	.001
	(.005)	(.005)	(.005)	(.096)	(.040)	(.066)	(.082)	(.076)	(.060)
1985	.182	.182	.182	.181	.183	.183	.181	.183	.183
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
1988	.180	.180	.181	.181	.181	.181	.181	.182	.183
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
1989	.171	.172	.172	.168	.170	.169	.169	.170	.171
	(.008)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
1992	.162	.162	.162	.159	.155	.159	.157	.160	.161
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
1994	.237	.236	.237	.250	.250	.251	.252	.253	.254
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
1996	.177	.177	.177	.176	.178	.177	.177	.177	.178
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
2000	.159	.159	.159	.159	.159	.159	.159	.159	.160
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
2004	.147	.148	.148	.133	.133	.134	.133	.134	.135
	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)
2007	.117	.117	.117	.115	.116	.116	.115	.117	.118
	(.003)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)	(.001)

Variance decomposition: Permanent and transitory effects

	Cross section	Decomposition	
		Perm. (%)	Trans. (%)
1977	.167	.033	.966
1982	.086	.507	.492
1987	.102	.624	.375
1992	.126	.709	.290
1997	.146	.769	.230
2002	.152	.823	.176
2007	.151	.886	.113
Mean	.129	.648	.351

Notes: Perm. stands for the share of cross sectional inequality due to the permanent heterogeneity components. Trans. stands for the share of cross-section inequality due to the transitory component.