Income Taxation in a Life Cycle Model with Human Capital

By

Michael P. Keane

University of New South Wales
Arizona State University

June 2009
Revised July 2011

Abstract: I examine the effect of labor income taxation in life-cycle models where work experience builds human capital. In this case, the wage no longer equals the opportunity cost of time – which is, instead, the wage *plus* returns to work experience. This has a number of interesting consequences. First, the data appear consistent with much larger labor supply elasticities than most prior work suggests. Second, again contrary to conventional wisdom, permanent tax changes can have larger effects on current labor supply than temporary tax changes. Third, human capital amplifies the labor supply response to permanent tax changes in the long-run, as a permanent tax reduces the rate of human capital accumulation (reducing worker productivity). Fourth, for plausible parameter values, welfare losses from proportional income taxation are likely to be much larger than conventional wisdom suggests.

Acknowledgements: This paper was presented as the Cowles Lecture at the 2011 North American Summer Meetings of the Econometric Society. Seminar participants at the University of Minnesota, the Australian National University, the University of Melbourne, the University of Zurich, the World Congress of the Econometric Society (Shanghai), the 2010 Australian Conference of Economists, the University of California at Berkeley, the 2011 SETA Conference (Melbourne) and the Federal Reserve Bank of New York provided useful comments. I thank Susumu Imai and Nada Wasi for performing many of the simulation exercises reported here. This research has been support by Australian Research Council grant FF0561843, by the AFTS Secretariat of the Australian Treasury and the ARC Centre of Excellence in Population Aging Research (grant CE110001029). But the views expressed are entirely my own.

I. Introduction

This paper examines the effects of income taxation in a life-cycle model where work experience builds human capital. In such a model, the wage no longer equals the opportunity cost of time. This has important implications for how workers respond to tax changes, and for how we ought to estimate labor supply elasticities. For instance, contrary to conventional wisdom, permanent tax changes can have larger effects on current labor supply than transitory ones. Thus, the Frisch (or transitory) elasticity may not be an upper bound on the Hicks and Marshallian. Furthermore, the human capital mechanism can greatly amplify the effect of permanent tax changes over time. As a result, studies that focus on short-run effects of tax reforms may greatly understate long-run labor supply elasticities.

Of course, Imai and Keane (2004) already studied introduction of human capital into the standard life-cycle model of MaCurdy (1981). They showed that ignoring human capital leads to severe downward bias in estimates of the intertemporal elasticity of substitution. But Imai-Keane failed to examine effects of permanent tax changes – i.e., Hicks and Marshallian elasticities. These are more relevant for tax policy. Here, I show how ignoring human capital can also lead one to understate elasticities with respect to permanent tax changes.

I will examine the implications of human capital for permanent and transitory taxes using both the Imai-Keane model and a simple two-period model. The former exercise has the advantage of "realism" – i.e., the Imai-Keane model provides a good fit to complete life-cycle paths of wages, hours and assets, so its' quantitative predictions about tax effects have some credibility. The two-period model has the virtue that it delivers intuitive analytical expressions for elasticities. This enhances our intuition for what drives the results, such as the conditions for permanent tax changes to have larger current effects than transitory changes.[1]

Using the simple model, I show that permanent tax changes have larger current effects than transitory changes under a condition that requires the returns to work experience to be sufficiently large relative to the size of the income effect. In a calibrated version of the simple two-period model, I show this does in fact occur for plausible parameter values.

Simulations of the Imai-Keane model imply that compensated permanent tax changes do have larger effects on current labor supply than transitory changes, but only for young workers (under 35) and older workers (over 50). This is consistent with the theory, as returns to experience are large for the young, while income effects are small for the old.

[1] The Imai-Keane model is rather complex, allowing for a very flexible human capital production function, wage uncertainty, taste shocks, a 45-period working life, a bequest motive, etc. These are all needed to provide a good fit to life-cycle wage, hours and asset paths. But this complexity precludes obtaining analytical results, and may make the intuition behind some results less clear.

One key experiment sheds light on the dynamics of tax effects: I use the Imai-Keane model to simulate a permanent tax increase that takes effect at age 25 and lasts for the whole working life (i.e., to age 65). Interestingly, the compensated elasticity grows from only 0.54 at age 25 to 4.0 at age 60. Thus, the human capital mechanism amplifies effects of permanent tax changes over time (i.e., higher taxes reduce current labor supply, leading to lower wages next period, etc.). If tax effects grow substantially over time, looking at short run effects of tax reforms may lead us to greatly understate labor supply elasticities in the long run.

Averaged over the whole life, the Hicks elasticity implied by the Imai-Keane model is a substantial 1.32. This, combined with the negative effect on human capital accumulation, generates substantial efficiency losses from distortionary proportional income taxation.

These findings are in sharp contrast to the consensus of the prior literature, which is based mostly on static models or dynamic models that include savings but not human capital. The consensus is summed up nicely in the survey by Saez, Slemrod and Giertz (2011), who state: "… optimal progressivity of the tax-transfer system, as well as the optimal size of the public sector, depend (inversely) on the compensated elasticity of labor supply …. With some exceptions, the profession has settled on a value for this elasticity close to zero… In models with only a labor-leisure choice, this implies that the efficiency cost of taxing labor income … is bound to be low as well."[2] The results presented here challenge this consensus by showing that, once we consider human capital, the data appear consistent with higher labor supply elasticities, and larger welfare losses from taxation, than is widely supposed.

To proceed, Section II presents a simple two-period version of the basic life-cycle model of labor supply and savings (MaCurdy (1981)). Section III discusses extension of the model to include human capital. Sections IV and V present simulations of both the two-period model and the Imai-Keane model, to see how the introduction of human capital alters the impact of tax changes. Section VI presents welfare calculations. Section VII concludes.

II. A Simple Life-Cycle Model without Human Capital

I start by presenting a simple model of life-cycle labor supply of the type that has strongly influenced economists' thinking on the subject since the pioneering work by MaCurdy (1981). In order to clarify the key points, it is useful to consider only two periods, and to abstract from wage uncertainty. The period utility function is given by:

[2] As Ballard and Fullerton (1992) note, if a wage tax is used to finance compensating lump sum transfers (as in the Harberger approach), the welfare cost depends only on the compensated elasticity. But if it is used to finance a public good (that has no impact on labor supply) it is the uncompensated elasticity that matters. Saez (2001) presents optimal tax rate formulas for a Mirrlees (1971) model (with both transfers and government spending on a public good) and shows that, in general, both elasticities matter for optimal tax rates (see, e.g., his equation 9).

(1) $$U_t = \frac{C_t^{1+\eta}}{1+\eta} - \beta \frac{h_t^{1+\gamma}}{1+\gamma} \qquad t = 1,2 \qquad \eta \le 0,\ \gamma \ge 0$$

Here C_t is consumption in period t and h_t is hours of labor supplied in period t. The present value of lifetime utility is given by:

(2) $$V = \frac{[w_1 h_1(1-\tau_1)+b]^{1+\eta}}{1+\eta} - \beta \frac{h_1^{1+\gamma}}{1+\gamma} + \rho \left\{ \frac{[w_2 h_2(1-\tau_2)-b(1+r)]^{1+\eta}}{1+\eta} - \beta \frac{h_2^{1+\gamma}}{1+\gamma} \right\}$$

Here w_1 and w_2 are wage rates in periods 1 and 2, while τ_1 and τ_2 are tax rates on earnings. Agents can borrow/lend across periods at interest rate r. The quantity b is net borrowing at t=1, and $b(1+r)$ is repaid at t=2. Parameter ρ is the discount factor. (I assume there is no exogenous non-labor income. This simplifies the analysis while not changing the results).

In the standard life cycle model, there is no human capital accumulation via returns to work experience. That is, hours of work in period 1 do not affect the wage rate in period 2, and a worker simply treats the wage path $\{w_1, w_2\}$ as exogenously given. Thus, the first order conditions for his/her optimization problem are simply:

(3) $$\frac{\partial V}{\partial h_1} = \left[w_1 h_1(1-\tau_1)+b\right]^{\eta} w_1(1-\tau_1) - \beta h_1^{\gamma} = 0$$

(4) $$\frac{\partial V}{\partial h_2} = \left[w_2 h_2(1-\tau_2)-b(1+r)\right]^{\eta} w_2(1-\tau_2) - \beta h_2^{\gamma} = 0$$

(5) $$\frac{\partial V}{\partial b} = \left[w_1 h_1(1-\tau_1)+b\right]^{\eta} - \rho\left[w_2 h_2(1-\tau_2)-b(1+r)\right]^{\eta}(1+r) = 0$$

Equation (5) can be written as $[C_1]^{\eta} \big/ [C_2]^{\eta} = \rho(1+r)$ – i.e., set b to equate the ratio of the marginal utility of consumption across the two periods to $\rho(1+r)$. Using (5), divide (4) by (3) and take logs to obtain MaCurdy's equation for hours changes as a function of wage changes:

(6) $$\ln\left(\frac{h_2}{h_1}\right) = \frac{1}{\gamma}\left\{ \ln\frac{w_2}{w_1} + \ln\frac{(1-\tau_2)}{(1-\tau_1)} - \ln \rho(1+r) \right\}$$

From (6) we obtain:

(7) $$\frac{\partial \ln(h_2/h_1)}{\partial \ln(w_2/w_1)} = \frac{1}{\gamma}$$

Thus, the intertemporal (or Frisch) elasticity of substitution, the rate at which a worker shifts hours of work from period 1 to period 2 as the relative wage increases in period 2, is simply $1/\gamma$. The elasticity with respect to a change in the tax ratio $(1-\tau_2)/(1-\tau_1)$ is identical.

While the Frisch elasticity is useful for predicting effects of transitory tax changes, the Hicks (compensated) and Marshallian (uncompensated) are relevant for predicting effects of permanent tax changes. In the life-cycle model (with no exogenous non-labor income) the Marshallian elasticity is $(1+\eta)/(\gamma-\eta)$ while the Hicks is $1/(\gamma-\eta)$. Thus, as long as $\eta < 0$, we have the famous relationship $1/\gamma > 1/(\gamma-\eta) > (1+\eta)/(\gamma-\eta)$. That is, the Frisch exceeds the Hicks which exceeds the Marshallian. The implication is that transitory tax changes will have larger (current) effects than permanent ones. [However, as we'll see, the Frisch does not give the exact effect of transitory taxes, as these may still have small income effects].

I'll now derive the exact elasticities with respect to permanent and transitory tax changes. In what follows I assume $\rho(1+r)=1$, so that (5) requires the consumer to equate the marginal utility of consumption in both periods. As the simple model in (1) has time invariant preferences, this is equivalent to equalizing consumption across periods. None of the key points I wish to make hinge on this assumption, and it simplifies the analysis considerably.

From (3) we have that:

$$\text{(8)} \qquad \frac{\beta h_1^{\gamma}}{C_1^{\eta}} = w_1(1-\tau_1)$$

where $C_1=w_1h_1+b$ is consumption in period 1. This is the familiar within-period optimality condition equating the marginal rate of substitution (MRS) between consumption and leisure to the opportunity cost of time, which is just the after tax wage rate. Given $\rho(1+r)=1$, we have $C_1=C_2=C$, and C is just the present value of earnings times the factor $(1+r)/(2+r)$:

$$\text{(9)} \qquad C = \{w_1(1-\tau_1)h_1(1+r) + w_2(1-\tau_2)h_2\}/(2+r)$$

Now we use equation (6) in levels, with $\rho(1+r)=1$, to substitute out for h_2 in (9), obtaining:

$$\text{(10)} \qquad C = h_1C^* = h_1\{w_1(1-\tau_1)(1+r) + w_2(1-\tau_2)\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1)}\right]^{1/\gamma}\}/(2+r)$$

Here C^* contains all the factors that govern lifetime wealth. We can now write (8) as:

$$\text{(11)} \qquad \ln h_1 = \frac{1}{\gamma-\eta}\left\{\ln w_1(1-\tau_1) - \ln\beta + \eta\ln C^*\right\}$$

We are now in a position to consider effects of permanent vs. temporary changes in tax rates. I'll start with uncompensated effects. Via some tedious algebra applied to (10)-(11) we can obtain the effect of an uncompensated tax reduction in period 1:

(12) $$\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \left[\frac{1+\eta}{\gamma-\eta}\right] - \left[\frac{\eta}{\gamma-\eta}\frac{1+\gamma}{\gamma}\frac{1}{1+x}\right] \quad where \quad x \equiv \left[\frac{w_1(1-\tau_1)}{w_2(1-\tau_2)}\right]^{(1+\gamma)/\gamma}(1+r)$$

Note that the first term on the right is the Marshallian elasticity. The second term is positive because $\eta<0$, so the elasticity with respect to a temporary tax change exceeds the Marshallian. If $w_1=w_2$ and $\tau_1=\tau_2$ then the second term in (12) takes on a simple form:

(13) $$\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \left[\frac{1+\eta}{\gamma-\eta}\right] - \left[\frac{\eta}{\gamma-\eta}\frac{1+\gamma}{\gamma}\frac{1}{2+r}\right]$$

Now consider a permanent tax change. We assume that $\tau_1 = \tau_2 = \tau$, and look at the effect of a change in (1- τ). With $\tau_1 = \tau_2 = \tau$ equation (10) becomes:

(10') $$C = h_1(1-\tau)C^{**} = h_1(1-\tau)\left\{w_1(1+r) + w_2\left[\frac{w_2}{w_1}\right]^{1/\gamma}\right\}/(2+r)$$

And we can rewrite (11) as:

(14) $$\ln h_1 = \frac{1}{\gamma-\eta}\left\{\ln w_1(1-\tau) - \ln\beta + \eta\ln(1-\tau)C^{**}\right\}$$

It is then clear that:

(15) $$\frac{\partial \ln h_1}{\partial \ln(1-\tau)} = \frac{1+\eta}{\gamma-\eta}$$

which is just the Marshallian elasticity. So, comparing (13) and (15), we have the well-known result that the uncompensated labor supply elasticity with respect to a temporary tax change is greater than that with respect to a permanent change in the standard life-cycle model. This is because the transitory tax has a smaller income effect, as captured by the 2nd term in (13).[3]

Now consider compensated tax changes. We can show that the compensated elasticity with respect to a transitory tax change (see equation (39) below) is given by:

(16) $$\left.\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)}\right|_{compensated} = \frac{1}{\gamma}\frac{1}{2+r} + \frac{1}{\gamma-\eta}\frac{1+r}{2+r}$$

Note that this falls between the Frisch and Hicks elasticities. Of course, the effect of a

[3] Another interesting question is whether an uncompensated transitory tax change has a larger effect than a compensated permanent change. This is ambiguous, as it depends on the size of income vs. inter-temporal substitution effects, and on the duration of the tax increase. In our case, equation (13) will exceed the Hicks elasticity if $(1+\gamma)/\gamma(2+r) > 1$, or equivalently if $0 < \gamma < (1+r)^{-1}$. So, as we'll see in the simulations, the outcome will depend on calibrated parameter vales.

compensated permanent tax change is the Hicks elasticity itself. Thus, we have the basic result that a compensated transitory tax change must have a larger (current) effect than a compensated permanent tax change. Of course, this is due to intertemporal substitution.

That transitory changes in taxes or wages should have greater effects on labor supply than permanent changes is firmly entrenched as the conventional wisdom in the profession. But in the next Section I show how introduction of human capital into the standard life-cycle labor supply model undermines this conventional wisdom, so that permanent tax changes can have larger effects than temporary changes (for a range of reasonable parameter values).

III. Incorporating Human Capital in the Life-Cycle Model

III.A. A Life-Cycle Model with Human Capital and Borrowing Constraints

I will begin by introducing human capital into a simple model with no borrowing or lending. This makes the impact of human capital on labor supply decisions clear. Assume that the wage in period 2, rather than being exogenously fixed, is an increasing function of hours of work in period 1. For convenience, here I assume the simple functional form:

(17) $$w_2 = w_1(1+\alpha h_1)$$

where α is the percentage growth in the wage per unit of work. Given a two period model with each period corresponding to 20 years, it is plausible in light of existing estimates that αh_1, the percent growth in the wage over 20 years, is on the order of 1/3 to 1/2.[4] Note that we could approximate (17) by $\ln W_2 \approx \ln W_1 + \alpha h_1$. Thus, it is similar to a conventional log wage function, but without the usual quadratic in hours. I introduce that in the simulation section, but for purposes of obtaining analytical results (17) is much more convenient.

In a model with human capital but no borrowing/lending, equation (2) is replaced by:

(18) $$V = \frac{[w_1 h_1(1-\tau_1)]^{1+\eta}}{1+\eta} - \beta\frac{h_1^{1+\gamma}}{1+\gamma} + \rho\left\{\frac{[w_1(1+\alpha h_1)h_2(1-\tau_2)]^{1+\eta}}{1+\eta} - \beta\frac{h_2^{1+\gamma}}{1+\gamma}\right\}$$

and the first order conditions (3)-(5) are replaced by:

(19) $$\frac{\partial V}{\partial h_1} = \left[w_1 h_1(1-\tau_1)\right]^{\eta} w_1(1-\tau_1) - \beta h_1^{\gamma} + \rho\left[w_1(1+\alpha h_1)h_2(1-\tau_2)\right]^{\eta} w_1\alpha h_2(1-\tau_2) = 0$$

(20) $$\frac{\partial V}{\partial h_2} = \left[w_1(1+\alpha h_1)h_2(1-\tau_2)\right]^{\eta} w_1(1+\alpha h_1)(1-\tau_2) - \beta h_2^{\gamma} = 0$$

[4] For instance, using the PSID, Geweke and Keane (2000) estimate that for men with a high school degree, average earnings growth from age 25 to 45 is 33% (most of which is due to wage growth). For men with a college degree the estimate is 52%. They also find that earnings growth essentially ceases after about age 45.

It is useful to rewrite (19) in the form:

$$(21)\qquad \frac{\beta h_1^{\gamma}}{C_1^{\eta}} = w_1(1-\tau_1) + \rho\left[\frac{C_2^{\eta}}{C_1^{\eta}}\right]\{w_1 \alpha h_2(1-\tau_2)\}$$

where $C_1=w_1h_1(1\text{-}\tau_1)$ and $C_2=w_1(1+\alpha h_1)h_2(1\text{-}\tau_2)$ are consumption at t=1 and 2.

One main point of this paper can be seen simply by comparing (8) and (21). Each equates the MRS to the opportunity cost of time. In the standard life-cycle model (8) this is just the after tax wage rate $w_1(1\text{-}\tau_1)$. But in the human capital model, the opportunity cost of time also includes a human capital investment component, which is $\rho[C_2^{\eta}/C_1^{\eta}]\{w_1\alpha h_2(1-\tau_2)\}$.

To understand this extra term, note that $dw_2/dw_1 = w_1\alpha$ is the effect of an additional hour of work a t=1 on the wage at t=2. This is multiplied by $h_2(1\text{-}\tau_2)$ to obtain the increment to after-tax *earnings*. It is also discounted back to t=1, and multiplied by the ratio of marginal utilities of consumption across periods (these may differ as there is no borrowing).

A key point that emerges from (21) is that a temporary tax change in period 1 affects only (1- τ_1), and hence it only affects the first component of the opportunity cost of time (the current wage rate). In contrast, a permanent tax change affects both (1- τ_1) and (1-τ_2), thus shifting both components of the opportunity cost of time (OCT). Thus, a permanent tax increase has a larger effect on the OCT than a temporary one.

In other words, the expectation of a lower future tax rate (at t=2) increases the return to human capital investment at t=1. This creates an extra incentive to work at t=1, in addition to that created by the lower tax at t=1 itself. Notably, this extra work incentive would be even greater in a model with multiple periods, as the future is then longer relative to the present.

We now derive elasticites with respect to uncompensated transitory and permanent tax cuts. To solve the model for h_1 we use (20) to solve for h_2 and substitute this into (19). This gives the following implicit function for h_1:

$$(22)\quad \beta h_1^{\gamma} = \left[w_1(1-\tau_1)\right]^{1+\eta} h_1^{\eta} + \rho\alpha\beta^{-(1+\eta)/(\gamma-\eta)}\left[w_1(1-\tau_2)\right]^{1+\eta+(1+\eta)^2/(\gamma-\eta)}(1+\alpha h_1)^{(1+2\eta+\gamma\eta)/(\gamma-\eta)}$$

As it is not possible to isolate h_1 in (22), we must totally differentiate to obtain the elasticity of hours in period 1 with respect to a transitory tax change – i.e., a change in (1-τ_1):

$$(23)\qquad \frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \frac{(1+\eta)[w_1(1-\tau_1)]^{1+\eta}h_1^{\eta}}{\gamma\beta h_1^{\gamma} - \eta[w_1(1-\tau_1)]^{1+\eta}h_1^{\eta} - \rho\alpha^2[w_1(1-\tau_2)]^{\Gamma_0}\beta^{-\Gamma_3}\Gamma_1(1+\alpha h_1)^{\Gamma_2}}$$

where $\Gamma_0 \equiv (1+\eta)(1+\gamma)/(\gamma-\eta)$, $\Gamma_1 \equiv (1+2\eta+\gamma\eta)/(\gamma-\eta)$, $\Gamma_2 \equiv (1+3\eta+\gamma\eta-\gamma)/(\gamma-\eta)$, $\Gamma_3 \equiv (1+\eta)/(\gamma-\eta)$.

Now consider the effect of a permanent tax increase. To simplify the analysis I will assume that $\tau_1 = \tau_2 = \tau$. This modifies (22) so that τ replaces that τ_1 and τ_2. As a result, when we totally differentiate (22) with respect to (1-τ) we get:

$$(24)\quad \frac{\partial \ln h_1}{\partial \ln(1-\tau)} = \frac{(1+\eta)[w_1(1-\tau)]^{1+\eta} h_1^{\eta} + \rho\alpha \dfrac{(1+\eta)(1+\gamma)}{\gamma-\eta}[w_1(1-\tau)]^{\frac{(1+\eta)(1+\gamma)}{\gamma-\eta}}(1+\alpha h_1)^{\Gamma_1}\beta^{-\Gamma_3}}{\gamma\beta h_1^{\gamma} - \eta[w_1(1-\tau)]^{1+\eta} h_1^{\eta} - \rho\alpha^2[w_1(1-\tau)]^{\Gamma_0}\Gamma_1(1+\alpha h_1)^{\Gamma_2}\beta^{-\Gamma_3}}$$

Of course (23) and (24) both simplify to the Marshallian elasticity $(1+\eta)/(\gamma-\eta)$ in the case of no human capital (α=0), as then we have a static model. Also, the denominators of (23) and (24) are identical.[5] The only difference between the two equations is an additional term in the numerator of (24) that captures an extra human capital effect of a permanent tax – i.e., a tax cut at t=2, by increasing the fraction of his/her earnings a worker can keep at t=2, increases the return to human capital investment (and so the opportunity cost of time) at t=1. Of course, the permanent tax also has an extra income effect – i.e., the worker knows he/she will receive a higher after-tax wage at t=2 even holding current hours fixed.

The sign of the extra term in the numerator of (24) depends on that of $(1+\eta)/(\gamma-\eta)$, the Marshallian elasticity itself. If the Marshallian elasticity is positive then a permanent tax cut has a larger positive effect on current labor supply than a transitory tax cut. If the Marshallian elasticity is negative, a permanent tax cut has a larger negative effect.

For compensated elasticities, the effect of permanent tax cuts must be positive and greater than that of transitory. This is because compensation eliminates the negative income effect of the t=2 tax cut on the incentive to invest in human capital at t=1.[6]

In summary, in a standard MaCurdy (1981)-style life-cycle model with borrowing but no human capital, the response to a temporary tax change is greater than to a permanent tax change – due to the income effect. But in the polar case of human capital but no borrowing, permanent taxes have a larger effect. In the next Section I present a model with both human capital and borrowing. Not surprisingly, whether permanent or temporary tax cuts have a larger effect will depend on the relative strength of the human capital and income effects.

[5] The addition of human capital adds a third term to the denominator of both (23) and (24). The fact that changes in hours at t=1 alter the wage at t=2 creates an incentive to invest, but also generates an income effect. Thus, the sign of the new term is ambiguous. It depends on $\Gamma_1 = (1+2\eta+\gamma\eta)$. For $-1 < \eta < -.5$ the transitory elasticity in (23) will actually be less than the Marshallian. This is possible because the transitory tax hits only a part of the OCT (i.e., the wage but not the human capital investment return). But if $-.5 < \eta < 0$ then, for plausible values of γ, the substitution effect is strong enough so that the transitory elasticity (23) is larger than the Marshallian. Of course, these considerations have no bearing on the relative size of the permanent and transitory elasticities in (23) vs. (24), as that depends only on the ratio of the numerators of the two equations.

[6] Indeed, this result only requires that compensation be adequate to hold consumption fixed at t=2, negating the income effect of the tax at t=2. Compensation at t=1 only affects the denominators of (23)-(24), not their ratio.

III.B. A Life-Cycle Model with both Human Capital and Saving/Borrowing

In a model with both human capital and borrowing/saving equation (2) is replaced by:

$$(25)\qquad V=\frac{[w_1h_1(1-\tau_1)+b]^{1+\eta}}{1+\eta}-\beta\frac{h_1^{1+\gamma}}{1+\gamma}+\rho\left\{\frac{[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)]^{1+\eta}}{1+\eta}-\beta\frac{h_2^{1+\gamma}}{1+\gamma}\right\}$$

and the first order conditions for the problem are:

$$(26)\qquad \frac{\partial V}{\partial h_1}=\left[w_1h_1(1-\tau_1)+b\right]^{\eta}w_1(1-\tau_1)-\beta h_1^{\gamma} + \rho\left[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)\right]^{\eta}w_1\alpha h_2(1-\tau_2)=0$$

$$(27)\qquad \frac{\partial V}{\partial h_2}=\left[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)\right]^{\eta}w_1(1+\alpha h_1)(1-\tau_2)-\beta h_2^{\gamma}=0$$

$$(28)\qquad \frac{\partial V}{\partial b}=\left[w_1h_1(1-\tau_1)+b\right]^{\eta}-\rho\left[w_1(1+\alpha h_1)h_2(1-\tau_2)-b(1+r)\right]^{\eta}(1+r)=0$$

As before, I assume $\rho(1+r)=1$ to simplify the analysis. In that case (26) can be rewritten:

$$(29)\qquad \frac{\beta h_1^{\gamma}}{C^{\eta}}=w_1(1-\tau_1)+\rho\alpha w_1h_2(1-\tau_2)$$

It is useful to compare this to (8), the MRS condition for the model without human capital. Here the opportunity cost of time is augmented by the term $\rho\alpha w_1h_2(1-\tau_2)$, which captures the effect of an hour of work at t=1 on the present value of earnings at t=2.

Now, continuing to assume $\rho(1+r)=1$, we divide (27) by (26) and take logs to obtain:

$$(30)\qquad \ln\left(\frac{h_2}{h_1}\right)=\frac{1}{\gamma}\ln\left[\frac{w_2(1-\tau_2)}{w_1(1-\tau_1)+\rho\alpha w_1h_2(1-\tau_2)}\right]$$

This equation illustrates clearly why the conventional procedure of regressing hours growth on wage growth leads to underestimates of the Frisch elasticity ($1/\gamma$), and overestimates of the key utility function parameter γ. The effective wage rate at t=1 is understated by failure to account for the term $\rho\alpha w_1h_2(1-\tau_2)$ that appears in the denominator.

We can get a better sense of the magnitude of the problem if we simplify by assuming $\tau_1=\tau_2=\tau$. Then we can solve (30) for $1/\gamma$ and obtain:

$$(31)\qquad \frac{1}{\gamma}=\ln\left(\frac{h_2}{h_1}\right)\div\ln\left(\frac{w_2}{w_1(1+\rho\alpha h_2)}\right)=\ln\left(\frac{h_2}{h_1}\right)\div\left[\ln\left(\frac{w_2}{w_1}\right)-\ln(1+\rho\alpha h_2)\right]$$

Thus, wage growth from t=1 to t=2 must be adjusted downward by a factor of $\rho\alpha h_2$ percent to correct for the missing human capital term. This adjustment gives a valid estimate of the growth of the opportunity cost of time (OCT).

As I noted earlier, a reasonable estimate of αh_1 is about 33%. A reasonable figure for hours growth over the first 20 years of the working life is roughly 20% (see, e.g., Imai and Keane (2004) or the descriptive regressions in Pencavel (1986)). So assume that h_2 is 20% greater than h_1. Then αh_2 is roughly 40%. Let $\rho = 1/(1.03)^{20} = 0.554$. Then we obtain $\rho\alpha h_2$ = 22%. Thus, while wage growth is 33%, the growth in the OCT is only 33% –22% = 11%.

Hence, if we used observed wage growth to calculate the Frisch elasticity we would obtain $(1/\gamma) = ln(1.20)/ln(1.33) \approx .64$. But the correct value based on equation (31) is $(1/\gamma)= ln(1.20)/ln[1.33/1.22] \approx 2.1$. So, for reasonable parameter values, the downward bias in estimates of the Frisch elasticity due to ignoring human capital will tend to be substantial.

Now consider the impact of uncompensated permanent vs. temporary tax changes in this model. First, solve (27) for h_2 to obtain:

$$(32) \qquad h_2 = \beta^{-1/\gamma}\left[w_1(1+\alpha h_1)(1-\tau_2)\right]^{1/\gamma} C^{\eta/\gamma}$$

Substituting this into (26) we obtain an implicit function for h_1:

$$(33) \qquad \beta h_1^{\gamma} = w_1(1-\tau_1)C^{\eta} + \rho\alpha\beta^{-1/\gamma} w_1^{(1+\gamma)/\gamma}(1+\alpha h_1)^{1/\gamma}(1-\tau_2)^{(1+\gamma)/\gamma} C^{\eta(1+\gamma)/\gamma}$$

Unfortunately (33) involves C, which is given by equation (9). Using (32) to substitute for h_2 in (9), we obtain an implicit function for C:

$$(34) \qquad C = \{w_1(1-\tau_1)h_1(1+r) + \left[w_1(1-\tau_2)(1-\tau_2)\right]^{(1+\gamma)/\gamma} \beta^{-1/\gamma} C^{\eta/\gamma}\}/(2+r)$$

We are now in a position to calculate labor supply elasticities of h_1 with respect to temporary tax changes, using the two-equation system (33)-(34). First, we implicitly differentiate (34) to obtain an expression for $dC/d(1-\tau_1)$ that involves $dh_1/d(1-\tau_1)$. Then we implicitly differentiate (33) to obtain an expression for $dh_1/d(1-\tau_1)$ that involves $dC/d(1-\tau_1)$. Finally, we substitute the former expression into the latter, group terms, and convert to elasticity form to obtain:

$$(35) \qquad \frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \frac{A\left[D + EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right] + \eta\left[A + B\frac{1+\gamma}{\gamma}\right]D}{\gamma\left[A - B\left(1+\frac{\alpha h_1/\gamma^2}{1+\alpha h_1}\right)\right]\left[D + EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right] - \eta\left[A + B\frac{1+\gamma}{\gamma}\right]\left[D + EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\frac{\alpha h_1}{1+\alpha h_1}\right]}$$

where:

$$A \equiv w_1(1-\tau_1)C^{\eta} \qquad B \equiv \rho\alpha\beta^{-1/\gamma}\left[w_1(1-\tau_2)\right]^{(1+\gamma)/\gamma}(1+\alpha h_1)^{1/\gamma}C^{\eta(1+\gamma)/\gamma}$$

$$D \equiv w_1 h_1(1-\tau_1)(1+r) \qquad E \equiv \beta^{-1/\gamma}\left[w_1(1-\tau_2)\right]^{(1+\gamma)/\gamma}(1+\alpha h_1)^{(1+\gamma)/\gamma}$$

The term B is the human capital affect that arises because an increase in h_1 leads to a higher wage at t+2. It is identical to the second term on the right side of (33). The $EC^{\eta/\gamma}(\gamma-\eta)/\gamma$ term is the usual income effect of the higher after-tax wage in period t=1. And the term $EC^{\eta/\gamma}[(1+\gamma)/\gamma][\alpha h_1/(1+\alpha h_1)]$ is a special income effect that arises because an increase in h_1 increases the wage rate at t=2. [Note that (35) reduces to (12) if we set α=0].

We now look at the effect of a permanent tax increase by setting $\tau_1 = \tau_2 = \tau$ in (33) and (34), and following the same solution procedure as above. This leads to the result:

$$\frac{\partial \ln h_1}{\partial \ln(1-\tau)} =$$

(36)
$$\frac{\left[A+\left\{B\frac{1+\gamma}{\gamma}\right\}\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right]+\eta\left[A+B\frac{1+\gamma}{\gamma}\right]\left[D+\left\{EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right\}\right]}{\gamma\left[A-B\left(1+\frac{\alpha h_1/\gamma^2}{1+\alpha h_1}\right)\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right]-\eta\left[A+B\frac{1+\gamma}{\gamma}\right]\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\frac{\alpha h_1}{1+\alpha h_1}\right]}$$

It can be shown that this reduces to the Marshallian elasticity if we set $\alpha = 0$. Compared to equation (35), equation (36) has two new terms, both of which appear in curly brackets in the numerator. The first is $\{B(1+\gamma)/\gamma\}$, which is an additional human capital effect. It captures that a lower tax rate in period t=2 increases the return on human capital investment at t=1. The second is $\{EC^{(\eta/\gamma)}(1+\gamma)/\gamma\}$ which captures an additional income effect (i.e., the lower tax in period 2 leads to higher lifetime income holding labor supply fixed).

Whether a permanent or a temporary tax change has a larger effect on labor supply depends on which of these two effects dominates. A permanent tax change will have the larger effect if the following condition holds:

$$\left\{B\frac{1+\gamma}{\gamma}\right\}\left[D+EC^{\frac{\eta}{\gamma}}\left(\frac{\gamma-\eta}{\gamma}\right)\right]>\eta\left[A+B\frac{1+\gamma}{\gamma}\right]\left\{EC^{\frac{\eta}{\gamma}}\left(\frac{1+\gamma}{\gamma}\right)\right\}$$

Some tedious algebra reveals that this condition is equivalent to a bound on the parameter α, which governs how work experience in period 1 affects the wage in period 2. The bound is:

$$(37) \qquad \alpha > \frac{-\eta\left(\beta h_1^{\gamma}\right)C^{-\eta}}{\rho(2+r)C+\eta h_1\left(\beta_1 h_1^{\gamma}\right)C^{-\eta}} > 0$$

Note that the numerator of (37) is obviously positive, as $\eta<0$, and the next two terms are the marginal utilities of leisure and consumption, respectively. But the sign of the denominator appears ambiguous, as the first term is positive while the second is negative. Appendix A shows that the denominator is positive under plausible conditions.

Thus, equation (37) gives a lower bound that the human capital effect α must exceed in order for (uncompensated) permanent tax changes to have larger effects than temporary tax changes in the model with human capital and saving. This lower bound is greater the stronger is the income effect. To see this, note that as η approaches -1 (i.e., income effects become stronger), the numerator of (37) increases while the denominator decreases. It is also obvious that when utility is linear in consumption (no income effects) (37) reduces to $\alpha > 0$.

If we make the approximation that $\alpha^2 \approx 0$, which is reasonable given that, as noted earlier, a plausible value for αh_1 is about .33, we can obtain the more intuitive expression:

$$(38) \qquad \alpha > -\left[\frac{\eta}{1+\eta}\right]\frac{w_1(1-\tau)}{w_1(1-\tau)h_1+\frac{1}{1+r}w_1(1-\tau)h_2}$$

This makes clear that the bound for α gets higher as income effects grow stronger. In the simulations of Section IV.C we will see clearly how the lower bound for α increases in ($-\eta$).

From a different perspective, these results say *human capital <u>dampens</u> the response to transitory tax changes*. Intuitively, this occurs because, if human capital is more important, a temporary tax hits a smaller part of the OCT – i.e., it only hits the wage and not the human capital return – so it has less effect on current labor supply. Hence, the simulations in Section IV.C will show that (35) is strongly *decreasing* in α. In contrast, we'll see that the effect of permanent taxes on current labor supply is little affected by α. This is because a permanent tax hits both current and future components of the OCT.[7] So, for large enough α, the effect of a permanent tax will be greater than that of a transitory tax.

Now consider <u>compensated</u> elasticities. First note there is no direct equivalent to the Slutsky equation in the dynamic case (i.e., as a value function rather than a utility function is being maximized). Thus, I have defined the compensated elasticity as the effect of a tax change holding the optimized value function fixed. In order to determine the amount of initial

[7] Mechanically, the human capital term B enters the numerator of (35) negatively, so the equation is decreasing in α. In contrast there are both positive and negative terms in B in equation (36), and these roughly cancel.

assets a consumer must be given to compensate for a tax change, I solve the equation:

$$(39) \qquad V(\tau_1, \tau_2, 0) = V(\tau_1', \tau_2', A) \approx V(\tau_1', \tau_2', 0) + u'(C)A \quad \Rightarrow \quad A \approx \frac{V(\tau_1, \tau_2, 0) - V(\tau_1', \tau_2', 0)}{u'(C)}$$

where τ_1' and τ_2' denote the tax rates after the tax change. Giving people the initial asset level defined by A in (39) equates the initial value function $V(\tau_1, \tau_2, 0)$ and the post-tax change value function $V(\tau_1', \tau_2', A)$ to a high degree of accuracy. For small tax changes this procedure is approximately equal to redistributing the proceeds lump sum.

We have already seen that permanent tax cuts can have larger effects on current labor supply in the uncompensated case, so this must be true a fortiori in the compensated case – i.e., compensation works to counteract the larger income effect of the permanent tax. Hence, I will not derive the result analytically. The simulations in Section IV show quantitatively how the bound that α must attain for permanent tax changes to have larger effects than transitory is smaller in the compensated case (i.e., smaller than the bound in (37)).

IV. Simulations of the Two-Period Model

IV.A. Two-Period Model Calibration

To calibrate the two-period model, we may think of each period as 20 years out of a 40-year working life (e.g., 25 to 44 and 45 to 64). I assume a real annual interest rate of 3%. As $1/(1+.03)^{20} = 0.554$, this implies a 20 year interest rate of $r = .806$. Thus, I assume the discount factor is $\rho = 1/(1+r) = 0.554$. I set the tax rates to $\tau_1 = \tau_2 = .40$ in the baseline model.

I will examine how the model behaves for a range of values of the key utility function parameters η and γ. Two (of the very few) studies that estimate life-cycle models with both savings and human capital, and that also assume CRRA utility, are Keane and Wolpin (2001) and Imai and Keane (2004). They estimate $\eta \approx -.5$ and $\eta \approx -.75$, respectively.[8] Goeree, Holt and Palfrey (2003) present experimental evidence, as well as evidence from field auction data, in favor of $\eta \approx -.4$ to $-.5$. Bajari and Hortacsu (2005) estimate $\eta \approx -.75$ from auction data. Thus, I primarily use values of -.50 and -.75 for η.[9] I will also consider the $\eta = -1.0$ case that is widely used in macro models (i.e., U=log(C) so income and substitution effects cancel).

[8] The only other study I am aware of with human capital, saving and CRRA utility is Van der Klauuw and Wolpin (2008). They obtain $\eta \approx -1.60$, which is much lower than the values I assume. This may be because they look at people at or near retirement, while Keane and Wolpin (2001) and Imai and Keane (2004) use data on young men. Shaw (1989) was the first to estimate a dynamic model with both human capital and saving. But she used translog utility, so the estimates are not useful for calibrating (1).

[9] These values of η imply less curvature in consumption (more intertemporal substitution) than much of the prior literature on consumption Euler equations. Keane and Wolpin (2001, p. 1078) argue that prior work gave downward biased estimates of η because it ignored liquidity constraints. Without liquidity constraints one needs $\eta \ll 0$ to explain why youth with steep age-earnings profiles don't borrow heavily in anticipation of earnings growth.

Of course, the value of γ is the subject of controversy. Most estimates of the Frisch elasticity ($1/\gamma$) are quite small (see Keane (2011)). Two rare exceptions are French (2005), who obtains 1.33 for 60 year olds in the PSID, and Heckman and MaCurdy (1982) who obtain 2.3 for married women in the PSID. But most estimates are in the 0 to .50 range. At the same time, many macro economists have argued that values of 2 or greater are needed to explain business cycle fluctuations using standard models (see Prescott (1986, 2006)).

Imai and Keane (2004) is a major exception to the prior literature, as they estimate that $\gamma \approx .25$. They were first to estimate the Frisch elasticity in a model that includes human capital. And they argue, for reasons similar to those discussed here, that failure to account for human capital led prior work to severely understate ($1/\gamma$).[10] Keane and Rogerson (2010) discuss a variety of other mechanisms that may have caused past work to understate ($1/\gamma$).

Given the controversy over γ, I examine the behavior of the model for a wide range of values. Specifically, I look at $\gamma = \{0, 0.25, 0.50, 1, 2, 4\}$. But I often focus on $\gamma = 0.50$, which I consider plausible in light of Imai and Keane (2004) and Keane and Rogerson (2010).

The β is just a scaling parameter that depends on the units for hours and consumption, and has no bearing on the substantive behavior of the model. Thus, in each simulation, I set β so optimal hours are 100 in the model without human capital (α=0). The initial wage w_1 is also set to 100. These values were chosen purely for ease of interpreting the results.

Finally, consider the wage function. In contrast to the simple function assumed for analytical convenience in Section III (equation 17), here I assume the more realistic:

$$(40) \qquad w_2 = w_1 \exp(\alpha h_1 - \kappa\left(h_1^2/100\right) - \delta)$$

This is similar to a standard Mincer log earnings function $\ln w_2 = \ln w_1 + \alpha h_1 - \phi\left(h_1^2/100\right) - \delta$, where w_1 is the initial skill endowment, and there is a quadratic in hours (experience). But I also include a depreciation term δ, which causes earnings to fall if the person does not work sufficient hours in period one (Keane and Wolpin (1997) find depreciation is important).

Given that β is chosen so hours will be close to 100 in period one, we can think of h=100 as corresponding (roughly) to full-time work and h=50 to part-time work.[11] I decided to calibrate the model so that: (i) a person must work at least part-time at t=1 for the wage not to fall at t=2, and (ii) the return to additional work falls to zero at 200 units of work. Given

[10] It is notable that French (2005), who also obtained a high value of ($1/\gamma$), did so for 60 year olds. As both Shaw (1989) and Imai and Keane (2004) note, human capital investment is less important for people late in the life-cycle. For them, the wage is close to the OCT, so the bias from ignoring human capital will be much less severe.

[11] Actually, agents will typically supply somewhat more than 100 units of labor when α>0, due to the incentives to acquire human capital.

these constraints, the wage function reduces to:

$$(41) \qquad w_2 = w_1 \exp\left(\alpha h_1 - \frac{\alpha}{4}\left(h_1^2/100\right) - \frac{175}{4}\alpha \right)$$

Thus, the single parameter α determines how work experience maps into human capital. I calibrate α so it is roughly consistent with the 33% to 50% wage growth for men that occurs from age 25 to age 45. As we'll see below, this requires α in the .008 to .010 range. However, I will also consider a range of other α values, to learn how the behavior of the model changes when human capital is more or less important.

IV.B. Baseline Simulation of the Two-Period Model

Table 1 reports baseline simulations of models with η = -.75 and η = -.50. It reports units of work in periods 1 and 2, as well as the wage rate in period 2. As the wage at t=1 is normalized to 100, one can read the amount of wage growth directly from the table. Results are reported for values of α ranging from 0 to .012. Recall that β is normalized in all models so h = 100 in the α=0 case. Thus, the overall level of hours rises as we move down the rows of the table and the return to human capital investment increases.

I first look at how the models capture wage growth. Consider models with η = -.50. At α = .008, wage growth ranges from 31% when γ = 4 to 46% when γ = 0, including 39% at my preferred value of γ = .50. These are solidly in the range of values that Geweke-Keane obtained from the PSID. At α = .010, wage growth ranges from 41% when γ = 4 to 66% when γ = 0, including 54% at my preferred value of γ = .50. This brings us to the upper end of the range of values that Geweke-Keane estimated. Based on these simulation results, I conclude that values of α in the .008 to .010 range are reasonable (at least for η = -.50).[12]

A notable feature of Table 1 is that wage growth is not very sensitive to the value of η. Thus, α = .008 to .010 is also a reasonable range when η = -.75.

Next consider how the models capture hours growth. Like wages, hours have a hump shape over the life cycle; but hours grow less rapidly at young ages. For instance, as Imai and Keane (2004) note, for men in the NLSY79, average annual hours increase 12% from age 25 to age 35. Descriptive hours regressions in Pencavel (1986) show a similar pattern. For the 1956-65 birth cohort (males and females), McGrattan and Rogerson (1998) use U.S. Census data to project average weekly hours of 30.8, 34.5, 34.2 and 18.6 at ages 25-34, 35-44, 45-54 and 55-64 (see their Table 10). Note that hours grow by 12% from the first interval (25-35) to the second (35-44). They then plateau before falling by 47% at ages 55-64.

[12] With α = .007, wage growth from t=1 to t=2 is 32% at my preferred value of γ = .50. This is plausible, but a bit low compared to the 33% to 52% values that Geweke and Keane (2000) estimated from the PSID.

Of course, a two-period model cannot capture the inverted-U shape of hours, as one would need at least three periods to capture both the growth at young ages and the decline at ages 55-64.[13] Thus, I calibrate the two-period model to capture the modest hours growth of roughly 10% to 15% that occurs over the life-cycle from youth to middle age.

In Table 1 we see that the model with α = .008, η = -.50, and γ = .50 (which I view as plausible values) gives an increase in work units from 121 at t=1 to 133 at t=2, meaning that hours growth is 10%. Thus, this model can be viewed as successfully capturing the modest growth in hours that we see for workers from their 20s to their 40s.

Most other specifications in Table 1 also appear reasonable, but a few can be ruled out. In particular, for α in the plausible .008 to .010 range, we see that models with γ = 0 generate implausibly large increases in labor supply (e.g., 69% in the α = .008, η = -.50 case).

Finally, I repeated the experiments described below in 3 and 4-period models, letting β increase in the last period to replicate the hours decline at ages 55-64. The results are quite similar (available on request). Furthermore, Section V simulates taxes in the full Imai-Keane (2004) model, which provides a good fit to hours, wages and assets over the whole life cycle (ages 20-65). The multi-period model gives new insights, but does not alter the key results.

IV.C. Effects of Taxes in the Two-Period Model

In this Section I use the simple two period model of Section III to simulate effects of temporary and permanent tax changes. All simulations assume that taxes have no effect on pre-tax wages. This is restrictive, but consistent with widely used macro models that assume a constant-returns-to-scale Cobb-Douglas production technology.

Tables 2 presents results for η = -.75 (the Imai-Keane estimate). The left panel shows elasticities with respect to temporary tax changes at t=1. The right panel shows elasticities with respect to permanent tax changes (i.e., changes that apply in both periods). The first four rows show results for α = 0, the case of no human capital accumulation. Lower rows show results for progressively higher values of α. The columns correspond to different values of γ.

Consider first the case of α = 0 and γ = .50, which is a common value in calibrating real business cycle models. Then, the Marshallian elasticity is $(1+\eta)/(\gamma-\eta)$ = 0.20, the Hicks (compensated) elasticity is $1/(\gamma-\eta)$ = 0.80, and the Frisch elasticity is $1/\gamma$ = 2.0. As we see in the first three rows of Table 2, the *theoretical* values of these elasticities correspond closely to the numerical values obtained by simulating the model. Slight differences arise only because we are taking finite difference derivatives (i.e., we increase $(1-\tau)$ by 1%).

[13] Also, to capture the ≈50% decline in average hours that occurs at ages 55-64, one would want to take account factors like declining tastes for work, health issues, pensions, etc.. The 2-period model abstracts from these issues, which are often captured by an age varying β in multi-period labor supply models (see Keane (2011)).

The Hicks and Marshall elasticities appear in the right panel of Table 2, as they are responses to permanent tax changes. The Frisch elasticity does not properly belong in either panel, as it measures hours growth in response to wage growth,[14] which is not identical to the response to a transitory tax change. The Frisch appears in the left panel only for convenience.

A key point is that elasticities with respect to transitory wage/tax changes at *t*=1do not correspond to any of the usual Marshall, Hicks of Frisch concepts. For example, we can use equation (13) to obtain the theoretical value of the uncompensated labor supply elasticity with respect to a temporary tax change at *t*=1 in the model with no human capital:

$$\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \left[\frac{1-.75}{0.50+.75}\right] - \left[\frac{(-.75)}{.50-(-.75)}\frac{1+.50}{.50}\frac{1}{2+.806}\right] = 0.20 + 0.64 = 0.84$$

This aligns closely with the value of 0.835 obtained in the simulation. This uncompensated elasticity actually exceeds the Hicks (0.80) for reasons discussed earlier (see footnote 3). Finally, Table 2 also reports a compensated elasticity with respect to a temporary tax cut of 1.222. This is also close to the theoretical value from equation (16), which is 1.228.

As expected, without human capital, the uncompensated and compensated elasticities with respect to transitory tax cuts greatly exceed those with respect to permanent tax cuts (e.g., 0.835 vs. 0.199 and 1.222 vs. 0.792, respectively, in the γ = .50 and η = -.75 case).

IV.C.1. The Effect of Small Returns to Work Experience on Frisch Elasticity estimates

The second panel of Table 2 presents results when the human capital effect is set at the very low level of α = .001. Strikingly, even this small value renders the conventional method of estimating the Frisch elasticity – i.e., taking the ratio of hours growth to wage growth – completely unreliable.[15] For instance, with γ = 0.50 and tax rates fixed, the wage increases by 3.25% from *t*=1 to *t*=2. At the same time, hours increase from 101.43 to 102.16, or 0.72%. Taking the ratio, we would infer a Frisch elasticity of only (1/γ)=0.72/3.25= 0.221, compared to the true value of (1/γ) = 2.0. [Note: Here I define the “correct” Frisch elasticity as the response of hours to changes in the *opportunity cost of time,* as in (31)].

One might surmise that the conventional method of calculating the Frisch elasticity is severely downward biased because the wage change from *t*=1 to *t*=2 in the baseline model (with a fixed tax rate of 40%) is entirely endogenous. It results entirely from human capital

---

[14] For example, in the α=0 panel of Table 2, in the γ = .50 case, the Frisch elasticity is reported as 2.01beacuse the wage grows by 1% between t=1 and t=2 and hours grow by 2.01%.

[15] Of course, econometric studies that estimate the Frisch elasticity by regressing hours changes on wage changes use more complex IV techniques, designed to deal with measurement error in wages, heterogeneity in tastes for work, and unanticipated wage changes. We do not have any of those problems here, so the appropriate estimator boils down to just taking the ratio of the percentage hours change to the percentage wage change.

investment. There is no source of exogenous variation in the after-tax wage, such as an exogenous tax change or a change in the rental rate on human capital (e.g., a labor demand shift). One might further surmise that if the data contained an event such as a temporary tax cut that shifted the wage path exogenously, one could infer γ more reliably.

Surprisingly, this intuition is fundamentally flawed. The row labelled "Frisch–tax" in Table 2 reports Frisch elasticities calculated in the conventional manner in a regime with a temporary 1% tax cut in t=1. Looking at the γ = 0.50 case, we see the estimate is -.478, which is not even the correct sign. Why? The tax cut causes labor supply to increase in period 1, which, in turn, increases the wage in period 2. But despite the wage increase, labor supply actually *declines* in period 2. This is because (i) the tax cut is removed, and (ii) the human capital investment part of the OCT is removed. Thus, although the wage is higher at t=2, the opportunity cost of time is lower. This illustrates well the important distinction between the wage and the opportunity cost of time in a model with human capital.

Another way to look at this is that a strictly exogenous shift in the wage path cannot exist in a model with human capital. For instance, a higher after-tax wage at t=1 increases hours, but this raises the wage at t=2 via the human capital effect. So a t=1 tax cut does not cause an exogenous change in the wage profile: the wage at t=2 is altered by the *behavioral response*. This has fundamental implications for estimation of wage elasticities. If experience alters wages, methods that rely on exogenous wage variation will not work. One must model the joint wage/labor supply process, and determine how labor supply responds to the OCT.

IV.C.2. Tax Effects with Plausible Returns to Work Experience (α = .008 to .010)

Now consider the case of α = .008. If we continue to focus on η = -.75 and γ = 0.50, then the baseline model (i.e. no tax change) generates 35.24% wage growth from t=1 to t=2 (see Table 1). As I noted in Section IV.B, this is roughly consistent with observations. Labor supply grows by 6.23%. So by conventional methods, we would estimate a Frisch elasticity of only 6.23/35.25 = 0.177. If, instead of the baseline, we use the data that includes a tax cut at t=1, we would obtain 0.198.[16] In either case, the estimate is far too small (as $1/\gamma$ = 2.0).

Now consider labor supply elasticities with respect to transitory (t=1) tax cuts. These are reported in the left panel of Table 2. The first thing to note is that both uncompensated and *compensated* elasticities drop substantially when human capital is included in the model. For example, they fall from .835 and 1.222 in the no human capital case to .312 and .606 in

[16] Recall that for α = .001 conventional methods produced very different Frisch estimates depending on whether the data contain a tax change. But for larger values like α = .008 the estimates are quite close (although far too small in any case). This is because at larger values of α wage growth from period t=1 to t=2 is much greater, and this insures that the OCT does increase (despite the tax rate increase and human capital return drop) at t=2.

the α=.008 case (i.e., by more than a factor of two). As I noted in Section III.B, it is intuitive that human capital dampens labor supply responses to temporary tax cuts: if human capital is more important, a temporary tax hits a smaller part of the OCT.

Next, compare responses to temporary vs. permanent tax rate changes. When α=.008 and γ = 0.50, the *uncompensated* elasticity of labor supply at t=1 with respect to a temporary tax cut at t=1 is 0.312, while that with respect to a permanent tax cut is 0.176. This seems consistent with the conventional wisdom that temporary tax changes have larger effects than permanent ones. However, the *compensated* elasticity of labor supply at t=1 with respect to a temporary tax cut is 0.606, while that with respect to a permanent tax cut is greater, 0.698. So, at least for compensated tax changes, we see it is indeed possible for permanent changes to have larger effects than transitory changes at plausible parameter values. In the next section, we'll see this can be true for uncompensated elasticities as well (see Table 3).

Finally, in Table 2 we also see that the Frisch elasticity – as conventionally measured – is 3 to 4 times smaller than compensated elasticities for both permanent and temporary tax changes. This illustrates another key point: the generally low estimates of the Frisch elasticity in the literature should not be viewed as an upper bound on compensated elasticities.[17]

IV.C.2. The Case *of* η = -.50 (*the Keane-Wolpin* (2001) *estimate*)

Next I turn to Table 3, which gives results for models with η = -.5 (the Keane-Wolpin (2001) estimate). Focus again on the γ = 0.50 case. In the model *without* human capital (α=0), the uncompensated elasticity with respect to a temporary tax cut at t=1 is, as expected, almost exactly twice as large as that with respect to a permanent tax cut (1.03 vs. 0.50).[18] But with plausible returns to work experience (α = .008), the uncompensated elasticity with respect to a permanent tax cut is *greater* than that with respect to a temporary tax cut (0.445 vs. 0.420). If we move to the α=.010 case, which is towards the higher end of the plausible range, the difference grows even larger (0.424 vs. 0.327).[19]

Recall that in Table 2, where η = -.75, we found that permanent tax cuts could have larger effects than transitory in the compensated case. Here, with η = -.50, we see this can also happen in the uncompensated case. This is consistent with the theory in Section III.

---

[17] In fact, the Frisch elasticities do not even give upper bounds for uncompensated elasticities. E.g., in the α=.008, γ = 0.50 case the two methods of calculating the Frisch elasticity produce values of 0.177 and 0.198, while the uncompensated elasticity for a t=1 tax cut is 0.312.

[18] Recall from equation (13) that $\frac{\partial \ln h_1}{\partial \ln(1-\tau_1)} = \left[\frac{1-.50}{0.50+.50}\right] - \left[\frac{(-.50)}{.50-(-.50)}\frac{1+.50}{.50}\frac{1}{2+.806}\right] = 0.50 + 0.53 = 1.03 \cdot$

[19] Of course, if uncompensated elasticites of permanent tax cuts exceed those of transitory then this must be true a fortiori for compensated elasticities. For instance, in the α = .008 and γ = 0.50 case the compensated elasticity with respect to a permanent tax cut is 0.884, while that with respect to a temporary tax cut is only 0.661.

Recall from equation (37) that the hurdle the human capital effect (α) must exceed for permanent tax cuts to have larger *uncompensated* effects is increasing in ($-\eta$). This hurdle was not met in Table 2, but it is met in Table 3 (where ($-\eta$) is smaller).

These results illustrate a key point: for plausible parameter values – indeed for what I have argued in Section IV.A are the preferred range of values for α, η and γ – labor supply effects of permanent tax cuts can exceed those of temporary tax cuts in the life-cycle model with human capital. Consistent with the theory section, this is more likely if income effects are weaker (and hence more likely for compensated elasticities).

Finally, note that in the α=.008 case the conventional method of calculating the Frisch elasticity produces values of .304 and .256 (if the data do or do not contain a temporary tax cut, respectively). These estimates, typical of the low values in prior empirical work, imply values of γ of 3.3 to 3.9. Yet we know the true value is γ=0.50. Strikingly, these conventional Frisch elasticity estimates are even smaller than the <u>un</u>compensated elasticity with respect to a permanent tax cut (.445) and much smaller than the compensated elasticity (.884).

This illustrates another important point: The low estimates of the Frisch elasticity obtained in prior literature are consistent not only with large values of ($1/\gamma$), but also with quite large values for compensated and even <u>un</u>compensated elasticities. Hence, existing estimates of the Frisch elasticity that ignore human capital should not be viewed as upper bounds on either of these elasticities.

IV.C.3. The Case of Log Utility (Income and Substitution Effects Cancel**)**

The previous two sub-sections presented results for calibrated values for η such that substitution effects dominate income effects (η>-1). Those values (i.e., η=-0.5 and -.75) were chosen based on results in Imai and Keane (2004), Keane and Wolpin (2001) and several other micro data studies. But macro models often assume log(C) utility to generate balanced growth paths. Thus, it is also interesting to consider the case of η=-1.

To conserve space I only give an overview of the results. If γ=0.5 wage growth is 33%, 45% and 59% in the α=0.008, 0.010 and 0.012 cases, and hours growth is 4% to 5%. So these cases all generate plausible wage and hours patterns. Note that, with log utility, larger values of the human capital effect (up to α = 0.012) become plausible.

Of course, the uncompensated elasticity with respect to permanent tax cuts is always zero, as income and substitution effects cancel. The uncompensated elasticities of t=1 labor supply with respect to transitory (t=1) tax cuts are 0.24, 0.18 and 0.13 in the α=0.008, 0.010 and 0.012 cases, respectively. So, exactly as expected, with log utility transitory tax cuts must have larger effects than permanent tax cuts in the uncompensated case.

But in the compensated case this outcome is reversed. Compensated elasticities of t=1 labor supply with respect to transitory tax cuts are 0.56, 0.48 and 0.42 for the α=0.008, 0.010 and 0.012 cases, respectively. But for permanent tax cuts the figures are 0.57, 0.56 and 0.55. A similar pattern holds for all γ in the 0.25 to 4 range.[20] Thus, for plausible human capital effects, and for all plausible γ, permanent tax effects are larger than transitory tax effects.

IV.C.4. *Results for the Model with Liquidity Constraints*

Finally I consider the model with borrowing constraints (Section III.A). I focus on the η = -.50 case.[21] As we saw in Section III.A, in a model with human capital but no borrowing, the uncompensated labor supply elasticity with respect to permanent tax changes *must* exceed that for temporary changes. So the issue is only one of magnitudes:

For instance, in the α = .008 and γ = 0.5 case, the uncompensated elasticity with respect to a temporary tax cut is 0.345 while that with respect to a permanent tax cut is 0.469. Similarly, the compensated elasticity with respect to a temporary tax change is 0.687 while that with respect to a permanent tax change is 0.958. (The method of compensation in this case is discussed in Appendix B). While both uncompensated and compensated elasticities with respect to permanent tax changes always exceed those for temporary tax changes, the size of the difference grows with the importance of human capital effects.

Recall that, in the model *with borrowing* (Table 3) it was necessary to have $\alpha \geq .006$ in order for compensated elasticities with respect to permanent tax cuts to exceed those for temporary tax cuts, and to have $\alpha \geq .008$ to get this outcome for uncompensated elasticities. With borrowing constraints, we see that any positive α generates this outcome. Models with and without borrowing constraints are polar cases, with the "truth" presumably somewhere in between. If borrowing constraints are important, more modest returns to experience will be sufficient for permanent tax changes to have larger effects than temporary ones.[22]

[20] For example, if γ=2.0, which conforms closely with conventional wisdom, the compensated elasticity of t=1 labor supply with respect to transitory tax cuts is 0.27, 0.24 and 0.22 for the α=0.008, 0.010 and 0.012 cases, respectively. But for permanent tax cuts these figures are 0.32, 0.31 and 0.31. Thus, the permanent tax effects are 20% to 40% greater than transitory effects.

[21] I focus on this case because Keane and Wolpin (2001) estimated the only life-cycle model I am aware of with both human capital and liquidity constraints, and they obtained η = -.50. Note that they estimated the extent of liquidity constraints, and their estimates implied rather tight limits on uncollateralized borrowing.

[22] We can also ask what borrowing constraints imply about the Frisch elasticity. Of course with no borrowing or lending the inter-temporal substitution mechanism is completely shut down. If taxes are temporarily low at t=1, one can't heed the advice "when the sun shineth make hay" (Heywood (1547)) and save part of the earnings for t=2. Hence, the Frisch elasticity properly defined does not exist. It still makes sense, however, to ask what one would obtain for the Frisch elasticity (and what one would infer about γ) if one applied conventional methods in an environment with liquidity constraints (Domeij and Floden (2006) ask a similar question). Of course, for the case of no human capital (α = 0) one just obtains the Marshallian elasticity. But when human capital is included one typically obtains *negative* values. For example, if α=.008 and γ=0.5 the Frisch elasticity appears to be -.189. The estimate is negative because the OCT falls over time while the wage increases.

V. Simulations of the Imai-Keane Model (Short vs. Long Run Tax Effects)

Next I use the Imai and Keane (2004) model to simulate effects of various types of tax changes. To my knowledge, this is the only micro model that attempts to fit asset, hours and wage data over the whole working life.[23] Because it generates life-cycle paths from age 20 to 65, it can be used to simulate the long-run effects of permanent tax changes. This was not possible in the simple 2-period model of Sections III-IV. The basic setup is as follows:

The Imai-Keane model assumes the same utility function as MaCurdy (1981). There are annual decision periods from age 20 to 65. At age 65 agents must retire, and there is a terminal value function that depends on assets (to generate a motive for retirement savings). The model contains a complex human capital production function that generalizes the simple 2-period model in several ways. In particular, it accommodates complimentarity between the stock of human capital and hours of work in the production of skill. Such complimentarity is quite evident in the data. Production function parameters are also allowed to vary in a flexible way with education and age. Wages are stochastic, and wage shocks exhibit persistence over time. Agents can borrow/lend across periods, and there are age varying tastes.

The model is fit to white males from the NLSY79 born in 1958-65 and followed from age 20 to 36. Imai and Keane (2004) document that the model produces a good in-sample fit to assets, hours and wages, both in terms of typical age profiles and dynamics/transition rates (e.g., it captures observed persistence in individual wages quite accurately). It also generates reasonable out-of-sample forecasts past age 36. For instance, the model accurately predicts the substantial fall in hours at older ages – e.g., it predicts an hours decline from ages 45-54 to 55-64 of 53%.[24] This is close to the 47% figure for this cohort projected by McGrattan and Rogerson (1998).[25] Also, when simulated data from the Imai-Keane model is used to estimate conventional labor supply functions, it gives conventional (i.e., small) wage elasticities.

As the Imai-Keane model provides a good fit to wage, hours and asset patterns – both in and out-of-sample – it seems credible to use it to predict labor supply responses. It is important to note, however, that this exercise compliments, but does not substitute for, the results from the simple 2-period model: For example, say we find permanent tax cuts have larger effects than transitory in the Imai-Keane model. As the model is complex, the intuition

---

[23] Two other papers that fit assets, hours and wages are Van der Klauuw and Wolpin (2008) and Keane and Wolpin (2001). The former simulates behavior of older workers, while the later focuses on youth.

[24] A limitation of the Imai-Keane model is it assumes interior solutions for hours, so it cannot generate complete retirement prior to age 65. But this limitation should not be exaggerated. In the 2008 CPS, 70% of men aged 55-64 still worked, and 52% of men aged 62-64 still worked (see Purcell (2009)).

[25] Overall, the Imai-Keane model predicts average weekly hours (for white males) of 44.4, 48.9, 43.4 and 19.9 at ages 25-34, 35-44, 45-54 and 55-64. This is similar to what McGrattan and Rogerson (1998) project for all men in this cohort (see their Table 8), but the Imai-Keane hours profile is shifted up due to exclusion of minorities.

for why this occurs would not be as transparent as in the simple model. As we'll see, results from the 2-period model are in fact quite useful for understanding what drives several of the results in the multi-period model.

To proceed, Table 4 reports simulated effects of permanent and transitory tax increases, using the Imai-Keane model. The effects of transitory tax increases were already reported in Imai and Keane (2004). But the permanent tax simulations, which are more relevant for evaluating long-run changes in tax policy, are new.

More specifically, Table 4 reports effects of 5% tax increases. In the column labelled "transitory," the tax increase applies for one year at the indicated age. For example, at age 20, a temporary 5% tax increase reduces hours by 1.5%. This implies an elasticity of only 0.30. This is far smaller than one might expect, given that Imai-Keane estimate $(1/\gamma) = 3.8$. But we see that effects of transitory taxes grow substantially with age. For instance, at age 60, a 5% tax increase reduces hours by 8.6%, implying an elasticity of 1.7.

The intuition for these two results is clear from our earlier discussion of the 2-period model, particularly equation (29). Transitory taxes have relatively small effects at young ages because they only affect a part of the OCT (i.e., the current after-tax wage, and not the return to human capital investment). But, as workers age, the current wage makes up a larger share of the OCT, so the elasticity with respect to transitory taxes increases. Thus, the human capital mechanism dampens the response to transitory taxes, especially for young workers.

The last two columns of Table 4 report effects of permanent 5% tax increases, both uncompensated and compensated. The tax increase occurs (unexpectedly) at the indicated age and lasts until age 65. The Table reports only the effect on current labor supply in the year the tax increase is first implemented. Note that compensated effects are much larger than uncompensated – implying that income effects are important – except at older ages.

The key result in Table 4 is that, for younger workers, (compensated) permanent tax increases have larger effects on current labor supply than do transitory tax increases. For instance, consider a 5% tax increase that takes place at age 25. If it is transitory, hours fall by 1.8%. But if it is permanent and proceeds are distributed lump sum, hours fall by 2.7%. So at age 25, the permanent tax effect is 50% greater. By the mid-30s permanent and transitory tax effects are roughly equal. Only in the 40s do transitory tax effects become somewhat larger.

These results are again consistent with intuition from the simple 2-period model. Permanent tax cuts can have larger effects on current hours than transitory tax cuts because they hit both the wage and human capital terms in (29), while a transitory tax only hits the current wage. But as workers age the returns to work experience fall. Eventually, the human capital effect is too small relative to the income effect (i.e., the bound in (37) is no longer satisfied) and transitory taxes begin to have a larger current effect than permanent taxes.

Another interesting aspect of Table 4 is that, starting at age 50, both compensated and uncompensated effects of permanent tax changes on current labor supply begin to grow quite rapidly. For instance, for workers aged 20 to 40, compensated effects of a 5% permanent tax increase are only -2.3 to -3.2%. But at ages 55 to 60 these effects grow to -7.2% and -10.5%. Uncompensated effects grow even more dramatically, from -0.7% at age 40 to -9.4% at 60.

This pattern emerges because, as one nears the terminal period, income effects of permanent taxes become negligible. Indeed, at age 60, compensated and <u>un</u>compensated elasticities of permanent tax changes slightly exceed that with respect to transitory changes (i.e., 2.1 and 1.9 vs. 1.7). Thus, even though the human capital effect is weak at age 60, it exceeds the even weaker income effect. [Note that <u>un</u>compensated permanent tax effects never exceed transitory effects at younger ages; this outcome is theoretically possible, but the income effect is too strong for it to occur in practice].[26]

So far, I have only discussed effects of tax changes on *current* period hours. Table 5 examines the long-run impact of permanent tax increases. The Table considers a permanent (compensated) 5% tax increase that takes effect at either age 25, 30 or 35, and that remains in effect for the rest of a worker's life. I report how this alters a person's labor supply at 5-year intervals from age 25 to 65. For instance, say a 5% tax increase goes into effect unexpectedly when the worker is 25. Then, at age 25, his labor supply is reduced by 2.7%. But, at age 45 his hours are reduced by 5.1%, and at age 60 the reduction is 19.3%.

The effect of a permanent tax change grows with age for two reasons: First, as I've already noted, as workers get older, the after-tax wage makes up a larger fraction of the OCT, so a given tax has a larger direct effect. Second, a permanent tax hike produces a "snowball" effect: If a worker reduces his labor supply at time t, he will have less human capital at time t+1. This causes him to work even less at time t+1, leading to a lower wage at t+2, etc..

This "snowball" effect of taxes on pre-tax wages is also shown in Table 5. At first, tax effects on human capital are small, but they grow substantially with age. For instance, if a 5% tax increase is instituted when a worker is 25, then by age 40 his wage is reduced by 1.0%, but by age 55 his wage is reduced by 3.6%, and by age 65 the reduction is 11.6%. So in the long-run a permanent tax reduces the rate of human capital accumulation. This lowering of pre-tax wages creates an additional work disincentive, beyond the direct effect of the tax.

[26] I have also calculated compensated elasticities with respect to transitory tax changes. Because income effects of transitory tax changes are minor, the differences from the figures in Table 4 are negligible.

Thus, we see that the human capital mechanism amplifies the effect of permanent tax changes in the long run. This result has important implications for the growing literature that attempts to estimate labor supply elasticities by looking at responses to major tax reforms (see Saez et al (2011) or Keane (2011) for reviews). This literature adopts a difference-in-difference approach and generally focuses on short-run responses. The results presented here suggest that a short-run focus may cause one to seriously understate responses to tax reforms.

With this issue in mind, I examine how a permanent tax increase affects labor supply over the entire working life. That is, I simulate the impact of a permanent 5% tax hike that starts at age 20 and lasts through age 65. If the revenue is thrown away, average hours (from age 20 to 65) drop by 2%. If the revenue is redistributed lump sum, average hours drop 6.6%. These figures imply uncompensated and compensated elasticities with respect to permanent tax changes of 0.4 and 1.3, respectively. These values are much larger than the 0.14 and 0.64 values one would obtain if one only looked at short run impacts at age 20 (see Table 4).

Notably, the compensated elasticity implied by the Imai-Keane parameter estimates in a model *without* human capital (a la MaCurdy (1981)) is $1/(\gamma\text{-}\eta) = 1/(.262+.736) \approx 1.0$. Thus, the human capital mechanism and the "snowball" effect on wages in the multi-period model combine to amplify the compensated elasticity by 30% (from 1.0 to 1.3). This is in sharp contrast to the dampening of the transitory elasticity that we discussed earlier.

VI. Human Capital and the Welfare Losses from Taxation

Here I return to the simple 2-period model of Section IV to consider how introducing human capital in the life-cycle model affects welfare losses from labor income taxation. I will assume a flat rate income tax equal to τ in both periods. To discuss optimal taxation it is necessary to specify that the government provides a public good from which workers derive utility.[27] Let the quantity of the public good be denoted by *P*, and assume the government provides the same level of *P* in each period. Then the government budget constraint is:

$$(42) \qquad P+\frac{1}{1+r}P=\left\{w_1h_1\tau+\frac{1}{1+r}w_2h_2\tau\right\} \qquad \Rightarrow \qquad P=\tau\left\{w_1h_1+\frac{1}{1+r}w_2h_2\right\}\frac{1+r}{2+r}$$

Next we modify the value function in equation (25) to include a public good:

$$(43) \quad V=\lambda f(P)+\frac{[w_1h_1(1-\tau)+b]^{1+\eta}}{1+\eta}-\beta\frac{h_1^{1+\gamma}}{1+\gamma}+\rho\left\{\lambda f(P)+\frac{[w_2h_2(1-\tau)-b(1+r)]^{1+\eta}}{1+\eta}-\beta\frac{h_2^{1+\gamma}}{1+\gamma}\right\}$$

Here the term $\lambda f(P)$ is the utility workers derive from the public good. I'll consider specific

[27] As we have a representative agent model, the redistributive motive for taxation that is central to work in the tradition of Mirrlees (1971) and Sheshinski (1972) is not relevant here.

functional forms below. Given (43), the worker's first order conditions are:

(44) $$\frac{\partial V}{\partial h_1} = \lambda f'(P)\frac{dP}{dh_1}(1+\rho) + \left[w_1 h_1(1-\tau)+b\right]^{\eta} w_1(1-\tau) - \beta h_1^{\gamma} + \rho\left[w_2 h_2(1-\tau) - b(1+r)\right]^{\eta}(dw_2/dh_1)h_2(1-\tau) = 0$$

(45) $$\frac{\partial V}{\partial h_2} = \lambda f'(P)\frac{dP}{dh_2}(1+\rho) + \rho\left\{\left[w_2 h_2(1-\tau) - b(1+r)\right]^{\eta} w_2(1-\tau) - \beta h_2^{\gamma}\right\} = 0$$

(46) $$\frac{\partial V}{\partial b} = \left[w_1 h_1(1-\tau)+b\right]^{\eta} - \rho\left[w_2 h_2(1-\tau)+b(1+r)\right]^{\eta}(1+r) = 0$$

where, given the wage equation (41) we have $dw_2/dh_1 = \alpha - \alpha h_1/200$, and so:

$$\frac{dP}{dh_1} = \tau\left\{w_1 + \frac{h_2}{1+r}\frac{dw_2}{dh_1}\right\}\frac{1+r}{2+r} = \tau\left\{w_1 + \frac{h_2}{1+r}w_2\left(\alpha - \frac{\alpha}{200}h_1\right)\right\}\frac{1+r}{2+r} \qquad \frac{dP}{dh_2} = \tau\frac{w_2}{2+r}$$

There is also a first order condition describing the problem of the government:

(47) $$\frac{\partial V}{\partial \tau} = \lambda f'(P)(1+\rho)\frac{\partial P}{\partial \tau} + C_1^{\eta}\frac{\partial C_1}{\partial \tau} + \rho C_2^{\eta}\frac{\partial C_2}{\partial \tau} = 0$$

As before I assume $\rho(1+r)=1$ in order to simplify the problem and focus on the key issues. In this case, and assuming no borrowing constraints, we have $C_1 = C_2 = C$. Then, (47) reduces to simply $\lambda f'(P)(1+\rho)\left[\partial P/\partial\tau\right] + C^{\eta}(1+\rho)\left[\partial C/\partial\tau\right] = 0$, and, as $\partial P/\partial\tau = -\partial C/\partial\tau$, we just have:

(48) $$\lambda f'(P) = C^{\eta}$$

This says the benevolent government (or social planner) sets the tax rate so as to equate the marginal utility of private consumption to that of public good consumption.

Equations (42)-(48) describe a social planner version of the model where the planner, in deciding on hours, considers how increased labor supply leads to increased provision of the public good. I also consider a "free rider" version of the model where there are many identical workers, and each assumes his/her own actions have a trivial impact on pubic good (i.e., dP/dh =0). Then the first term in equations (44)-(46) drops out.

To complete the model we must specify the functional form of $f(P)$. The curvature of $f(P)$ determines how the relative size of the public sector changes as workers grow wealthier. I consider three alternative forms: $f(P) = \log(P)$, $f(P) = 2P^{.5}$ and $f(P) = P$. These correspond to cases where P/C declines, is stable or grows as C increases. One might try to calibrate $f(P)$ by looking at how public spending evolves as countries become richer. But my main results are not very sensitive to this assumption – at least qualitatively – so I do not pursue this.

All models are calibrated with η = -.50 or η = -.75. The scaling parameter λ is set so the optimal tax rate is 40% when there is no human capital accumulation (i.e., when α = 0). Table 6 shows how optimal income tax rates vary with γ and α, for the η = -.50 case. Optimal tax rates are quite similar in the η = -.75 case, so I do not report them.

In the left panel of Table 6, we have $f(P) = \log(P)$. Thus, utility has more curvature in the public good than the private good. Note that as α increases people become wealthier (because, *ceteris paribus*, their t=2 wage is higher). Thus, in the log(P) case, the optimal tax rate falls as α increases. For instance, if we increase α from 0 to .008, and adopt my preferred value of γ=.50, the optimal tax rate falls from 40% to 33.9% in the free rider model.

Next consider the case of $f(P) = 2P^{.5}$. Here (48) is simply $\lambda P^{-0.5}=C^{-0.5}$. As curvature of the utility function in the public and private goods is equal, the optimal rate is always 40%.[28]

The right panel of Table 6 reports results for the case of $f(P) = P$. Here (48) reduces to $\lambda = C^{\eta}$. So the government sets the tax rate to keep the marginal utility of private consumption constant. Hence, the optimal tax rate in increasing in α. For example, when α=.008 and γ=.50, the optimal tax rate is 51.9% in the free-rider version of the model.

Next, Table 7 reports welfare costs of proportional income taxes, for different values of the utility (γ and η) and human capital (α) parameters. I report results for the free-rider version of the model only. To measure welfare loss, I must also solve a version of the model where a lump sum tax is used to finance the public good. The lump sum tax is set to the level that funds the same level of P as in the proportional tax version of the model.

The first measure of welfare loss, denoted C^* in Table 7, is the extra consumption that must be given to workers in the proportional tax world to enable them to attain the same level of the optimized value function (see equation (39)) as in the lump-sum tax world, expressed as a fraction of consumption in the proportional tax world. The second measure, C^{**}, is the loss in consumption in the lump-sum world that brings the worker down to the utility level of the proportional tax world, expressed as a fraction of consumption in the lump-sum world.

Table 7 part A reports results for the $f(P)=\log(P)$ case. The left panel reports results for η = -.75 and the right panel reports results for η = -.50. Each panel gives results for γ equal to 0.50 or 4, and for α equal to 0 or 0.008.

The most interesting comparison is between the "conventional" case of (α = 0, γ = 4), which coincides with the typical results from prior studies that estimate the Frisch elasticity ignoring human capital, and the setting (α = .008, γ = 0.5), which I argue is plausible once

[28] As $C=I(1-\tau)$ and $P=I\tau$, where I is $(1+r)/(2+r)$ times the PV of lifetime income, we have $P/C = \tau/(1-\tau)$. This implies $\lambda^2=\tau/(1-\tau)$ so $\tau= \lambda^2 /(1+ \lambda^2)$. Thus the optimal tax rate is a constant that only depends on λ.

one accounts for human capital. In the "conventional" setting, welfare losses are only 3 to 4% of consumption, regardless of the welfare measure. This is consistent with the conventional wisdom that welfare losses from taxation are small. But in the (α = .008, γ = 0.5) case the welfare losses are much larger, ranging from 8.3% to 11.4% of consumption.[29]

By combining results from Tables 6 and 7, we can express these welfare losses as a fraction of revenue raised. This gives a more even comparison across scenarios that involve different tax rates. For the "conventional" (α = 0, γ = 4) case, taking b=C^* and using η = -.50, the welfare loss is only 5.8% of revenue. But for my preferred case of (α = .008, γ = 0.5) the loss is 21.5% of revenue.[30] Thus, the welfare loss as a fraction of revenue is 4 times greater when we use parameter values that account for human capital.

Next consider Part B of Table 7, which reports results for the case where $f(P) = 2P^{.5}$. As noted earlier, if η = -.50 the optimal tax is always 40%, while if η = -.75 the optimal tax is slightly increasing in α.[31] As before, in the "conventional" case (α = 0, γ = 4) welfare losses are only 3 to 4% of consumption. But with human capital (α = .008, γ = 0.5) welfare losses are much larger, ranging from 12 to 18% of consumption, depending on the measure used.

Now consider welfare losses as a percent of revenue. For the "conventional" (α=0, γ=4) case we again have 5.8%. (As the tax rate is unchanged and α=0 there is no reason for results to change). For my preferred case of (α=.008, γ=0.5, η = -.50) we have 27.2%. So, using parameter values that account for human capital, we obtain a utility cost to consumers about 4.5 times greater than in the "conventional" case.

Finally, Part C of Table 7 reports results for $f(P) = P$. As we saw in Table 6, in this case the optimal tax rate rises substantially with α (as people become wealthier they demand more of the public good). For example, if γ=0.5 the optimal rate is 51.9% when α=.008. As tax rates are high, so are the welfare losses from income taxation. For example, if α = .008, η= -.75, γ = 1/2, the welfare loss is 19 to 28% of consumption, depending on the measure.

We can again express the utility loss as a percent of revenue. For my preferred case of (α=.008, γ=0.5), and with η = -.5, we have 39.8%. This is 6.7 times greater than the cost calculated using "conventional" parameter values that ignore human capital.

---

29 The results are not very sensitive to η: In the η = -.75 case welfare losses are again only 3% to 4% if α = 0, rising to 9% to 11% if α = .008.

30 We obtain these figures as follows: Letting R denote revenue, we have $R = C \cdot \tau/(1\text{-}\tau)$, and letting W denote welfare loss, we have $W = b \cdot C$, where b is the percentage loss reported in Table 7. Thus, the welfare loss to workers as a fraction of revenue is $W/R = b \cdot \tau/(1\text{-}\tau)$. In the "conventional" (α=0, γ=4) case, using η = -.50 and taking b=C^*, we have W/R = (3.90) (1-.40)/(.40) = 5.8%. For my preferred case (α=.008, γ=0.5) we have W/R = (11.38)(1-.347)/(.347) = 21.5%. [It is appropriate to use $b = C^*$ in these calculations, as $R = C^* \cdot \tau/(1\text{-}\tau)$ is the actual revenue under the proportional tax].

31 For example, when γ = 0.5 the optimal tax rate increases to 42.7% when α increases to .008.

In summary, while results of welfare calculations differ in detail for different choices of $f(P)$, the basic pattern is similar: In a conventional parameterization of the life-cycle model that ignores human capital, welfare losses are only 3 to 4% of consumption and less than 6% of revenue raised. But if we use parameters that are plausible in a human capital version of the life-cycle model, welfare losses as a fraction of revenue are 4 to 7 times greater.

VII. Conclusion

If human capital is added to the standard life-cycle labor supply model, the wage no longer equals the opportunity cost of time (OCT). Rather, the OCT is the wage *plus* returns to work experience. This has important implications for how workers respond to taxes, and for proper estimation of labor supply elasticities. In fact, given human capital, the data appear consistent with much larger labor supply elasticities than conventional wisdom suggests.

Another key implication is that the human capital mechanism dampens responses to transitory tax changes. This is because a transitory tax only affects one part of the OCT, the current after-tax wage, while leaving the return to human capital investment unaffected.

In contrast, effects of permanent taxes on the OCT are amplified. This is because they alter both the current wage and the future return to human capital investment. As a result, it is possible for permanent taxes to have larger effects on *current* labor supply than transitory tax changes – contrary to conventional wisdom based on models without human capital.

The condition for permanent taxes to have larger effects than transitory is that returns to work experience must be sufficiently large relative to income effects. In a simple 2-period model, I showed this can happen for plausible parameter values. Then, using the multi-period labor supply model of Imai and Keane (2004), which gives a good fit to life-cycle paths of wages, hours and assets, I found that permanent tax changes are likely to have larger effects than transitory for workers under 35 (for whom returns to work experience are large), and for workers over 50 (for whom income effects of permanent tax changes are small).

As the Imai-Keane model generates complete life-cycle paths of worker behavior, it can also be used to simulate the long-run response to permanent changes in tax policy. The results indicate that human capital amplifies the response to permanent tax changes in the long-run. For example, consider a tax change that goes into effect at age 25 and lasts through age 65. The (compensated) elasticity of labor supply at age 25 is only 0.54. But the elasticity grows to 1.0 at age 45 and 3.9 at age 60.

The effect of permanent tax changes grows over time because of a "snowball" effect on human capital investment: A permanent tax increase at t leads to less labor supply at t, which lowers wages at t+1, further reducing labor supply at t+1, etc.. In fact, in the previous example, a 5% tax increase leads to a 7.5% reduction in human capital by age 60. This in turn causes labor supply to fall disproportionately at older ages.

If the effects of permanent tax changes grow substantially over time, it has important implications for the literature that attempts to estimate labor supply elasticities by looking at responses to major tax reforms (see Saez et al (2011) or Keane (2011) for reviews). This literature generally focuses on short-run responses. But the results presented here suggest that a short-run focus may cause one to seriously understate responses to tax reforms.

Another way to view this phenomenon is to note that the preference parameters in the Imai-Keane model imply a compensated (Hicks) elasticity of about 1.0 in a world with no human capital. But simulation of their model generates a compensated elasticity of only 0.54 at age 25 rising to 3.9 at age 60, with an average over the whole working life of 1.30. Thus, the impact on lifetime hours is 30% greater than if the human capital mechanism were shut down. Thus, not only does human capital cause permanent taxes to have a larger effect in the long-run (i.e., at older ages), it also amplifies their total impact on lifetime hours.

Another key result is that even a "small" return to work experience (in a sense made precise in the paper) can lead to severe downward bias in *conventional* methods of estimating the intertemporal (Frisch) elasticity – where, by "conventional," I mean methods that ignore human capital. I showed that use of exogenous tax regime changes to identify labor supply elasticities does not resolve the problem of bias in conventional estimation methods, and can even make the bias greater.[32]

I went on to use a simple 2-period model with a government provided public good to study welfare effects of proportional (i.e., flat-rate) income taxation. The most interesting comparison is between parameterizations consistent with: (i) estimates of "conventional" MaCurdy (1981)-style models that ignore human capital, and (ii) estimates of Imai and Keane (2004)-style models, which include human capital. As expected, the conventional models imply welfare losses from taxation that are less than 6% of revenue raised – consistent with the conventional wisdom (described in the introduction) that welfare costs of income taxation are small. But the human capital models imply welfare losses as a percent of revenue that are 4 to 7 times greater (depending on details of the model).

The large welfare costs of taxation found here arise for two reasons: The first and more obvious is that human capital models generate larger labor supply elasticities. The second and subtler reason involves dynamics: As I noted earlier, a permanent tax hike not

[32] The point is that tax changes change the incentives to acquire human capital. Thus, any change in the time path of after-tax wages induced by exogenous tax changes will nevertheless be endogenous – as the wage path is influenced by changes in human capital investment decisions. The only solution is to model the joint labor supply/human capital investment process, as in Heckman (1976) and Imai and Keane (2004).

only reduces labor supply, but also the rate of human capital accumulation, reducing worker productivity. This suggests that static models that focus on the effect of taxes *holding work experience fixed* are missing a key channel through which welfare costs of taxation arise.

Three limitations of this paper are worth noting: First, the Imai-Keane model used here assumes interior solutions for hours. [This is also true of MaCurdy (1981) and most of the male labor supply literature]. The model correctly generates that average hours fall by about 50% from ages 45-54 to ages 55-64. But it cannot generate that roughly 30% of males aged 55-64 do not work (see Purcell (2009)). Instead, the model approximates this by having some workers reduce hours to low levels at ages 55-64. Also, the model imposes retirement at age 65, rather than treating it as a choice. It is not clear if these simplifications would affect the main results presented here, but building corner solutions into the model is an important avenue for future research. However, it is usually argued that corner solutions lead to higher labor supply elasticities (see below). And my results suggest tax increases have larger effects at older ages, so presumably they would generate more early retirement. Hence, it seems unlikely that including corners and/or retirement decisions would alter the main message of the paper – i.e., that labor supply elasticities are larger than conventional wisdom suggests.

Second, the models presented here ignore schooling, and consider the behavior of workers conditional on their having entered the labor force. But as noted by Keane and Wolpin (2000, 2010), changes in the tax/transfer system that reduce rewards to working will also reduce educational attainment. So accounting for this additional channel would presumably magnify the long-run tax effects on human capital found here.

Third, I assume a particular investment mechanism (learning-by-doing) while others may be operative – e.g., on-the-job training (OJT). But OJT models are similar to my model in key respects: The observed wage (i.e., earnings/hours) differs from the OCT (productivity) as only a fraction of work hours are spent in production. The rest is spent learning. Learning time falls with age, so *wages grow more slowly than the OCT*. This is the exact same problem that causes labor supply elasticities to be underestimated if we ignore learning-by-doing.

This paper is part of an emerging literature exploring mechanisms that may have caused prior work to understate labor supply elasticities. Besides the human capital, other potentially important mechanisms include liquidity constraints (Domeij and Floden (2006)), uninsurable wage risk (Low and Maldoom (2004)), corner solutions (Rogerson and Wallenius (2007), French (2005), Kimmel and Kniesner (1998)) and fixed costs of adjustment (Chetty (2010)). An important task for future research is to assess the relevance of these mechanisms. Suffice it to say, while conventional wisdom says labor supply elasticities are small, more dissent from that position is emerging – see Keane and Rogerson (2010) for a detailed survey.

Appendix A: The Sign of the Denominator of Equation (37)

We can show the denominator of (37) is positive as follows: Utilizing that $\rho(1+r)=1$, so that $\rho(2+r)=(1+\rho)$, we see that, in order for the denominator to be positive, we must have:

$$\text{(A1)} \qquad C > \frac{-\eta}{1+\rho} h_1 \frac{\left(\beta h_1^{\gamma}\right)}{C^{\eta}}$$

Now recall from equation (29) that $\frac{\beta h_1^{\gamma}}{C^{\eta}} = w_1(1-\tau) + \rho\alpha w_1 h_2 (1-\tau)$. Thus we have that:

$$\text{(A2)} \qquad C > \frac{-\eta}{1+\rho} h_1 \left[w_1(1-\tau) + \rho\alpha w_1 h_2 (1-\tau)\right] = \frac{-\eta}{1+\rho}\left[w_1 h_1 (1-\tau) + \frac{1}{1+r}(\alpha h_1) w_1 h_2 (1-\tau)\right]$$

where in the second term on the right we have substituted $\rho(1+r)=1$. Of course we have that the present value of lifetime consumption equals that of lifetime income:

$$\text{(A3)} \qquad \frac{2+r}{1+r} C = \left\{ w_1(1-\tau)h_1 + \frac{1}{1+r} w_1(1+\alpha h_2)(1-\tau) h_2 \right\}$$

Thus, the term in the square brackets in (A2) is $\frac{2+r}{1+r}C - \frac{1}{1+r} w_1 h_2 (1-\tau)$, which is lifetime income minus a part of period 2 earnings. So we can rewrite (A2) as:

$$\text{(A4)} \qquad C > \frac{-\eta}{1+1/(1+r)}\left[\frac{2+r}{1+r}C - \frac{1}{1+r} w_1 h_2 (1-\tau)\right] = -\eta\left[C - \frac{1}{2+r} w_1 h_2 (1-\tau)\right]$$

As long as $\eta > -1$ (i.e., substitution effects dominate income effects) this inequality must hold. The right hand side takes on its greatest value when $\eta = -1$, and then (A4) just says that C is greater than a fraction of C. The case where this condition fails is actually uninteresting, because then the bound in (37) is negative, and it will be satisfied by any positive α.

Appendix B: Compensation in the Liquidity Constrained Model

With liquidity constraints, we can no longer use (39) to determine how to compensate agents for a tax change, as now the marginal utility of consumption differs in the two periods. Thus, to compensate for a permanent tax change, I find the asset level A that solves:

$$\text{(B1)} \qquad A \approx \frac{V(\tau_1, \tau_2, 0) - V(\tau_1', \tau_2', 0)}{\left[u'(C_1) + u'(C_2)\right]/2}$$

and give the agent $A(1+r)/(2+r)$ in each period. To compensate for a temporary tax change in period 1, I find the asset level A that solves:

$$\text{(B2)} \qquad A \approx \frac{V(\tau_1, \tau_2, 0) - V(\tau_1', \tau_2, 0)}{u'(C_1)}$$

and give the agent A in period 1.

References

Bajari, P. and A. Hortacsu (2005). Are Structural Estimates of Auction Models Reasonable? Evidence from Experimental Data, *Journal of Political Economy*, 113:4, 703-741.

Ballard, C. and D. Fullerton (1992). Distortionary Taxes and the Provision of Public Goods, *Journal of Economic Perspectives*, 6:3, 117-131.

Blundell, R. and T. MaCurdy (1999). Labor Supply: A Review of Alternative Approaches. In Handbook of Labor Economics, Vol. 3, Ashenfelter and Card (eds.), 1559-95.

Chetty, Raj (2010). Bounds on Elasticities with Optimization Frictions: A Synthesis of Micro and Macro Evidence on Labor Supply. Working paper, Harvard University.

Domeij, D. and M. Floden (2006). The Labor Supply Elasticity and Borrowing Constraints: Why Estimates are Biased, *Review of Economic Dynamics*, 9, 242-262.

Eckstein, Z. and K.I. Wolpin. (1989). Dynamic Labor Force Participation of Married Women and Endogenous Wage Growth. *Review of Economic Studies*, 56: 375-90.

French, E. (2005). The Effects of Health, Wealth and Wages on Labour Supply and Retirement Behaviour. *The Review of Economic Studies*, 72, 395-427.

Geweke, J. and M. Keane (2000). An Empirical Analysis of Male Income Dynamics in the PSID: 1968-1989. *Journal of Econometrics*, 96: 293-356.

Goeree, J.K., C.A. Holt and T.R. Palfrey (2003). Risk Averse Behavior in Generalized Matching Pennies Games, *Games and Economic Behavior*, 45:1, 97-113.

Heckman, J. (1976). "Estimates of a Human Capital production Function Embedded in a Life-Cycle Model of Labor Supply," in N. Terlecky, ed., Household Production and Consumption (New York: Columbia University Press). 227-264.

Heckman, J.J and T. E. MaCurdy (1982). Corrigendum on a life cycle model of female labor supply. *Review of Economic Studies*, 49, 659-660.

Heywood, John. (1547). *A Dialogue contayning in effect the number of al the proverbes in the English tongue, compact in a matter concerning two marriages.*

Imai, S. and M. Keane (2004). Intertemporal Labor Supply and Human Capital Accumulation. *International Economic Review*, 45:2, 601-642.

Keane, M. (2011). Labor Supply and Taxes, *Journal of Economic Literature*, forthcoming.

Keane, M. and R. Rogerson (2010). Reconciling Micro and Macro Labor Supply Elasticities, Working paper, Arizona State University.

Keane, M. P. and K. I.Wolpin (2010). The Role of Labor and Marriage Markets, Preference Heterogeneity, and the Welfare System in the Life Cycle Decisions of Black, Hispanic and White Women. *International Economic Review*, 51:3, 851-892.

Keane, M. and K. Wolpin (2001). Effect of Parental Transfers and Borrowing Constraints on Educational Attainment. *International Economic Review*, 42:4, 1051-1103.

Keane, M. and K. Wolpin (2000). Equalizing Race Differences in School Attainment and Labor Market Success. *Journal of Labor Economics*, 18:4, 614-52.

Keane, M. and K. Wolpin (1997). The Career Decisions of Young Men. *Journal of Political Economy*, 105:3.

Kimmel J. and Knieser T.J. (1998). New evidence on labor supply: employment vs. hours elasticities by sex and marital status. *Journal of Monetary Economics*, 42, 289-301.

Low, Hamish and D. Maldoom (2004). Optimal Taxation, Prudence and Risk-Sharing, *Journal of Public Economics*, 88, 443-464.

MaCurdy, T. (1981). An Empirical Model of Labor Supply in Life-cycle Setting. *Journal of Political Economy*, 88, 1059-1085.

McGrattan, E. and R. Rogerson (1998). Changes in Hours Worked Since 1950. *Federal Reserve bank of Minneapolis Quarterly Review*, Winter, 2-19.

Mirrlees, J. (1971). An Exploration in the Theory of Optimal Income Taxation, *The Review of Economic Studies*, 38, 175-208.

Pencavel, John (1986), “Labor Supply of Men: A Survey,” in O. Ashenfelter and R. Layard (eds.), *Handbook of Labor Economics*, Vol. 1, North-Holland, Amsterdam, pp. 3-102.

Prescott, E. (1986). Theory Ahead of Business Cycle Measurement. *Federal Reserve Bank of Minneapolis Quarterly Review*, Fall, 9-22.

Prescott, E. (2006). Nobel Lecture: The Transformation of Macroeconomic Policy and Research, *Journal of Political Economy*, 114:2, 203-235.

Purcell, P. (2009). Older Workers: Employment and Retirement Trends, Congressional Research Service, Washington, D.C., Available at: www.crs.gov.

Rogerson, R. and J. Wallenius (2009). Micro and Macro Elasticities in a Life Cycle Model with Taxes, *Journal of Economic Theory*, 144, 2277-2292.

Saez, Emmanuel. (2001). Using Elasticities to Derive Optimal Income Tax Rates, *The Review of Economic Studies*, 68, 205-229.

Saez, E., J. Slemrod and S. Giertz (2011). The Elasticity of Taxable Income with Respect to Marginal Tax Rates: A Critical Review. *Journal of Economic Literature*,forthcoming.

Shaw, Katherine (1989). Life-cycle Labor Supply with Human Capital Accumulation. *International Economic Review*, 30 (May), 431-56.

Sheshinski, E. (1972). The Optimal Linear Income Tax. The Review of Economic Studies, 39:3, 297-302.

Van der Klaauw, W. and K.I. Wolpin (2008). Social Security and the Retirement and Savings Behavior of Low Income Households. *Journal of Econometrics*,145, 21-42.

Table 1: Baseline Simulation

α	**γ**	**η = -.50 (Keane-Wolpin)** 0	0.25	0.5	1	2	4	**η = -.75 (Imai-Keane)** 0	0.25	0.5	1	2	4
0	h_1		100	100	100	100	100		100	100	100	100	100
	h_2		100	100	100	100	100		100	100	100	100	100
	Wage(2)		100	100	100	100	100		100	100	100	100	100
0.001	h_1	101	103	102	101	101	100	99	102	101	101	101	100
	h_2	113	104	103	102	101	101	109	103	102	101	101	100
	Wage(2)	103	103	103	103	103	103	103	103	103	103	103	103
0.003	h_1	109	109	107	105	103	102	104	105	105	103	102	101
	h_2	135	115	110	106	103	102	120	110	107	104	103	101
	Wage(2)	111	111	111	111	110	110	111	111	111	110	110	110
0.005	h_1	120	116	112	108	105	103	110	109	108	106	104	102
	h_2	166	128	118	110	106	103	132	117	112	108	105	103
	Wage(2)	122	121	120	119	118	118	120	120	119	119	118	118
0.007	h_1	133	124	118	112	107	104	116	114	112	109	106	103
	h_2	209	145	128	116	108	104	147	126	118	111	106	104
	Wage(2)	137	134	132	129	127	126	131	130	129	128	127	126
0.008	h_1	140	128	121	114	108	105	120	117	114	110	107	104
	h_2	236	155	133	118	110	105	155	130	121	113	108	104
	Wage(2)	**146**	**141**	**139**	**135**	**133**	**131**	**138**	**137**	**135**	**133**	**132**	**130**
0.010	h_1	152	138	128	119	111	106	127	122	118	113	109	105
	h_2	299	178	146	125	113	106	174	141	128	117	110	105
	Wage(2)	**166**	**159**	**154**	**149**	**144**	**141**	**154**	**151**	**149**	**145**	**142**	**140**
0.012	h_1	162	147	136	124	114	107	135	128	123	117	111	106
	h_2	372	206	161	132	116	108	196	152	135	121	112	106
	Wage(2)	188	180	174	165	157	152	173	168	165	160	155	151

Table 2: Labor Supply Responses to Tax Changes, Case of η = -.75

		Tax reduction in period 1						Tax reduction in both periods					
		γ						γ					
α	**Elasticity**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	Total		1.570	**0.835**	0.445	0.235	0.122		0.249	**0.199**	0.142	0.090	0.052
	Compensated		2.059	**1.222**	0.721	0.410	0.223		0.990	**0.792**	0.566	0.361	0.209
	Frisch		4.060	2.010	1.000	0.499	0.249						
	Frisch – tax		4.060	2.010	1.000	0.499	0.249						
0.001	Total	7.784	1.278	0.733	0.408	0.220	0.116	0.212	0.236	0.194	0.140	0.090	0.052
	Compensated	8.192	1.731	1.104	0.675	0.392	0.215	0.841	0.935	0.770	0.558	0.358	0.208
	Frisch	3.200	0.430	0.221	0.109	0.053	0.025						
	Frisch – tax	-5.203	-0.839	-0.478	-0.263	-0.141	-0.074						
0.003	Total	2.267	0.891	0.572	0.341	0.192	0.103	0.223	0.220	0.186	0.137	0.089	0.052
	Compensated	2.663	1.297	0.917	0.596	0.357	0.200	0.883	0.874	0.739	0.546	0.354	0.207
	Frisch	1.404	0.390	0.208	0.099	0.045	0.020						
	Frisch – tax	0.814	0.197	0.086	0.027	0.003	-0.002						
0.005	Total	1.185	0.645	0.450	0.285	0.166	0.091	0.231	0.213	0.181	0.135	0.088	0.052
	Compensated	1.571	1.020	0.773	0.528	0.326	0.185	0.913	0.843	0.719	0.538	0.352	0.206
	Frisch	1.019	0.359	0.195	0.090	0.038	0.015						
	Frisch – tax	0.874	0.314	0.162	0.065	0.020	0.005						
0.007	Total	0.714	0.473	0.353	0.236	0.142	0.079	0.234	0.208	0.178	0.134	0.088	0.052
	Compensated	1.079	0.820	**0.657**	0.469	0.297	0.171	0.920	0.822	**0.705**	0.532	0.350	0.206
	Frisch	0.836	0.332	0.183	0.082	0.032	0.011						
	Frisch – tax	0.817	0.350	0.190	0.079	0.024	0.005						
0.008	Total	0.565	0.405	**0.312**	0.214	0.131	0.074	0.232	0.205	**0.176**	0.133	0.088	0.052
	Compensated	0.913	0.738	**0.606**	0.441	0.283	0.164	0.911	0.811	**0.698**	0.530	0.350	0.206
	Frisch	0.774	0.319	0.177	0.079	0.029	0.009						
	Frisch – tax	0.791	0.358	0.198	0.083	0.025	0.004						
0.010	Total	0.358	0.295	**0.241**	0.174	0.111	0.064	0.221	0.198	**0.173**	0.132	0.088	0.052
	Compensated	0.663	0.597	**0.515**	0.391	0.257	0.151	0.865	0.783	**0.683**	0.525	0.349	0.206
	Frisch	0.682	0.296	0.165	0.072	0.024	0.006						
	Frisch – tax	0.752	0.370	0.210	0.088	0.024	0.002						
0.012	Total	0.229	0.211	0.183	0.139	0.092	0.054	0.200	0.188	0.168	0.131	0.088	0.052
	Compensated	0.479	0.478	0.434	0.344	0.233	0.139	0.784	0.741	0.663	0.520	0.349	0.207
	Frisch	0.615	0.276	0.154	0.065	0.019	0.003						
	Frisch – tax	0.727	0.380	0.220	0.091	0.023	-0.001						

Note: η = -.75 is the Imai and Keane (2004) estimate. The "Total" elasticity is the uncompensated. The "Frisch" elasticity refers to the estimate obtained using the conventional method of regressing the log hours change on the log earnings change, using the simulated data. In the rows labelled simply "Frisch" the estimate is obtained from simulated data where the tax rate is equal in the two periods. In the rows labelled "Frisch – tax" the Frisch estimate is obtained using data that contain a tax cut at t=1. The figures in bold are obtained using values of the return to human capital investment in the plausible range (α = .008 to .010), and for my preferred value of γ=0.5.

Table 3: Labor Supply Responses to Tax Changes, Case of η = -.5

		Tax reduction in period 1						**Tax reduction in both periods**					
		γ						γ					
α	**Elasticity**	**0**	**0.25**	**0.5**	**1**	**2**	**4**	**0**	**0.25**	**0.5**	**1**	**2**	**4**
0	Total		1.844	**1.030**	0.568	0.305	0.160		0.666	**0.499**	0.332	0.199	0.111
	Compensated		2.279	**1.353**	0.783	0.434	0.231		1.326	**0.994**	0.663	0.398	0.221
			4.060	2.010	1.000	0.499	0.249						
	Frisch – tax		4.060	2.010	1.000	0.499	0.249						
0.001	Total	7.854	1.518	0.915	0.526	0.289	0.153	0.650	0.634	0.487	0.329	0.198	0.110
	Compensated	8.265	1.923	1.226	0.735	0.415	0.223	1.289	1.262	0.971	0.656	0.396	0.220
		3.865	0.480	0.238	0.114	0.054	0.026						
	Frisch – tax	-3.734	-0.713	-0.431	-0.249	-0.138	-0.073						
0.003	Total	2.306	1.083	0.732	0.451	0.257	0.139	0.700	0.599	0.472	0.324	0.197	0.110
	Compensated	2.703	1.447	1.022	0.651	0.379	0.207	1.384	1.191	0.940	0.646	0.393	0.220
		2.064	0.503	0.251	0.112	0.048	0.021						
	Frisch – tax	1.502	0.346	0.147	0.046	0.009	-0.001						
0.005	Total	1.176	0.795	0.589	0.386	0.228	0.125	0.710	0.579	0.462	0.321	0.196	0.110
	Compensated	1.541	1.127	0.861	0.578	0.347	0.192	1.399	1.149	0.920	0.640	0.392	0.220
		1.686	0.509	0.256	0.110	0.043	0.016						
	Frisch – tax	1.438	0.493	0.242	0.092	0.028	0.006						
0.007	Total	0.654	0.583	0.472	0.329	0.202	0.113	0.641	0.552	0.452	0.319	0.196	0.110
	Compensated	0.945	0.878	0.724	0.513	0.316	0.177	1.259	1.095	0.898	0.635	0.392	0.220
		1.536	0.507	0.257	0.107	0.038	0.013						
	Frisch – tax	1.334	0.553	0.287	0.114	0.034	0.007						
0.008	Total	0.489	0.495	**0.420**	0.303	0.189	0.107	0.578	0.532	**0.445**	0.318	0.197	0.110
	Compensated	0.732	0.768	**0.661**	0.482	0.302	0.171	1.134	1.054	**0.884**	0.633	0.392	0.220
		1.500	0.505	0.256	0.105	0.036	0.011						
	Frisch – tax	1.297	0.574	0.304	0.121	0.036	0.007						
0.010	Total	0.275	0.349	**0.327**	0.254	0.165	0.095	0.436	0.475	**0.424**	0.315	0.197	0.111
	Compensated	0.431	0.570	**0.541**	0.423	0.274	0.157	0.854	0.938	**0.841**	0.626	0.393	0.221
		1.470	0.502	0.254	0.102	0.032	0.008						
	Frisch – tax	1.246	0.610	0.334	0.134	0.038	0.005						
0.012	Total	0.162	0.240	0.249	0.210	0.143	0.084	0.315	0.400	0.390	0.309	0.197	0.111
	Compensated	0.255	0.407	0.431	0.367	0.248	0.144	0.618	0.789	0.774	0.614	0.393	0.222
	Frisch	1.470	0.501	0.251	0.098	0.029	0.005						
	Frisch – tax	1.213	0.642	0.362	0.145	0.039	0.003						

Note: η = -.5 is the Keane and Wolpin (2001) estimate**.** The “Total” elasticity is the uncompensated. The “Frisch” elasticity refers to the estimate obtained using the conventional method of regressing the log hours change on the log earnings change, using the simulated data. In the rows labelled simply “Frisch” the estimate is obtained from simulated data where the tax rate is equal in the two periods. In the rows labelled “Frisch – tax” the Frisch estimate is obtained using data that contain a tax cut at t=1. The figures in bold are obtained using values of the return to human capital investment in the plausible range (α = .008 to .010), and for my preferred value of γ=0.5.

Table 4: Labor Supply Responses to Different Types of Tax Increases in a Model with Human Capital (Imai-Keane)

	Transitory	Permanent (Unanticipated)	
Age		Uncompensated	Compensated
20	**-1.5%**	-0.7%	**-3.2%**
25	**-1.8%**	-0.6%	**-2.7%**
30	**-2.2%**	-0.6%	**-2.4%**
35	-2.6%	-0.5%	-2.3%
40	-3.2%	-0.7%	-2.3%
45	-3.8%	-1.0%	-2.8%
50	-4.7%	-2.3%	-4.2%
55	-6.2%	-5.3%	**-7.2%**
60	-8.7%	**-9.4%**	**-10.5%**

Note: All figures are contemporaneous effects of a 5% tax increase. The "transitory" increase only applies for one year at the indicated age. The "permanent" tax increases take effect (unexpectedly) at the indicated age and last until age 65. In the "compensated" case the proceeds of the tax (in each year) are distributed back to agents in lump sum form.

Table 5: Effects of Permanent Tax Increases on Labor Supply At Different Ages in a Model with Human Capital (Imai-Keane)

Age	Age 25			Age 30 (unexpected)			Age 35 (unexpected)	
	Hours	Wage		Hours	Wage		Hours	Wage
25	-2.7%							
30	-2.9%	-0.4%		-2.4%				
35	-3.2%	-0.7%		-2.7%	-0.3%		-2.3%	
40	-3.8%	-1.0%		-3.3%	-0.6%		-2.7%	-0.2%
45	-5.1%	-1.3%		-4.4%	-0.9%		-3.8%	-0.5%
50	-7.9%	-2.0%		-7.0%	-1.4%		-6.2%	-1.0%
55	-13.3%	-3.6%		-12.2%	-2.9%		-11.0%	-2.3%
60	-19.3%	-7.5%		-18.4%	-6.6%		-17.4%	-5.8%
65	-29.2%	-11.6%		-28.1%	-10.7%		-26.9%	-9.7%

Note: The tax increase is 5%. It takes effect (unexpectedly) at the indicated age and lasts until age 65. The proceeds of the tax (in each year) are distributed back to agents in lump sum form.

Table 6: Optimal Tax Rates (η = -.5)

		$f(P)$ = log(P)					$f(P) = P$			
		α					α			
γ		**0**	**0.007**	**0.008**	**0.010**		**0**	**0.007**	**0.008**	**0.010**
0.5	Social planner	0.400	0.368	0.362	0.349		0.400	0.646	0.680	0.746
	Free-rider	0.400	0.348	**0.339**	0.320		0.400	0.503	**0.519**	0.550
1	Social planner	0.400	0.375	0.370	0.361		0.400	0.548	0.571	0.616
	Free-rider	0.400	0.363	0.357	0.344		0.400	0.489	0.503	0.530
2	Social planner	0.400	0.380	0.377	0.369		0.400	0.498	0.513	0.543
	Free-rider	0.400	0.374	0.370	0.360		0.400	0.476	0.487	0.510
4	Social planner	0.400	0.383	0.381	0.375		0.400	0.473	0.484	0.506
	Free-rider	**0.400**	0.380	0.377	0.371		**0.400**	0.465	0.475	0.494

Note: The figures in bold correspond to the (α=0, γ=4) case, which conforms closely to the conventional wisdom for the value of γ in models without human capital, and the (α=.008, γ=0.5) case, which represents my preferred value based on estimates that account for human capital. These figures will be used later to calculate the welfare losses from taxation as a fraction of revenues.

Table 7: Welfare Losses from Proportional Income Tax

Case A: Utility from Public Good, f(P) = log(P)

η = -.75		$\alpha = 0$	$\alpha = .008$		η = -.50		$\alpha = 0$	$\alpha = .008$
γ= 0.50	C*	11.25	**11.42**		**γ= 0.50**	C*	15.56	**11.38**
	C**	-9.26	**-9.33**			C**	-10.74	**-8.29**
γ= 4.0	C*	**3.56**	3.87		**γ= 4.0**	C*	**3.90**	3.88
	C**	**-3.33**	-3.60			C**	**-3.51**	-3.48

Case B: Utility from Public Good, f(P) = $2P^{1/2}$

η = -.75		$\alpha = 0$	$\alpha = .008$		η = -.50		$\alpha = 0$	$\alpha = .008$
γ= 0.50	C*	11.25	**15.47**		**γ= 0.50**	C*	15.56	**18.11**
	C**	-9.26	**-12.06**			C**	-10.74	**-11.98**
γ= 4.0	C*	**3.56**	4.59		**γ= 4.0**	C*	**3.90**	4.62
	C**	**-3.33**	-4.23			C**	**-3.51**	-4.09

Case C: Utility from Public Good, f(P) = P

η = -.75		$\alpha = 0$	$\alpha = .008$		η = -.50		$\alpha = 0$	$\alpha = .008$
γ= 0.50	C*	11.25	**27.57**		**γ= 0.50**	C*	15.56	**41.62**
	C**	-9.26	**-19.09**			C**	-10.74	**-21.46**
γ= 4.0	C*	**3.56**	6.98		**γ= 4.0**	C*	**3.90**	7.88
	C**	**-3.33**	-6.24			C**	**-3.51**	-6.62

Note: C* = consumption gain needed to compensate for tax distortion
(starting from proportional tax world)
C** = equivalent consumption loss (moving from lump sum tax to distorting tax world)